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AN APPLICATION OF THE FINITE ELEMENT METHOD TO ELASTIC-PLASTIC PROBLEMS OF PLANE STRESS

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FOREWORD

This report was prepared by the Illinois Institute of Technology Research Institute and the Air Force Flight Dynamics Laboratory as a joint in-house effort under Project No. 1467, "Structural Analysis Methods."

The computer program presented here was developed through a number of modifications during the period January 1967 through December 1968.

This report has been reviewed and is approved.

A handwritten signature in dark ink, appearing to read 'Francis J. Janik, Jr.', with a large, stylized initial 'F' and 'J'.

FRANCIS J. JANIK, JR.
Chief, Solid Mechanics Branch
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ABSTRACT

A computer program is presented for the small strain analysis of plane structures in the strain hardening elastic-plastic range. The finite element displacement method is used to perform the linear analyses in the iterative scheme. Bar and constant strain isotropic plane stress triangles are available for use in constructing idealizations. The use of ten different sets of material properties, three different material laws, and incremental proportional loading are available as options. Good correlation is shown with available test data and theoretical solutions.

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SECTION I

INTRODUCTION

This report describes a computer program for the stress analysis of plane structures in the elastic-plastic range by the finite element method. The program can handle bar and triangular plate elements so that it is applicable to trusses and to the analysis of in-plane stresses in reinforced plates. The material behavior is assumed to be isotropic and the user has a choice of three types of stress-strain laws and ten different materials.

A numerical step by step procedure for obtaining solutions which satisfy the requirements of the incremental theory of plasticity for materials which obey the Mises yield condition and the associated flow rule is used in the program. At each step in the solution, an iterative procedure is used to find the correct values of the strain increments. Changes in plastic strain are accounted for by the addition of fictitious plastic forces to the actual loading on the structure in such a way that the deflections of the structure under the modified loading with assumed elastic behavior are equal to the actual deflections. A modified form of the computer program given in Reference 1 is used to obtain elastic solutions.

The finite element method originally developed for elastic stress analysis was extended to apply to inelastic material behavior by Padlog (Reference 2) et al. Further studies of the use of the method in elastic-plastic problems have been made, for example, at MIT (Reference 3) and the California Institute of Technology (Reference 4).

The method has been extended to apply to anisotropic materials by Jensen (Reference 5). Marcal and King (Reference 7) have applied the method to problems of plane stress and strain and to axisymmetric problems. However, the computer programs necessary for the application of the method have not been published. The purpose of this report is to make such a program available.

The following section of the report presents a detailed explanation of the method of analysis. The application of the program is demonstrated by examples in Section III. Directions for the use of the computer program are given in Section IV. Listing of the program and sample data sets are given in the Appendix.

$$A^{-1} = \frac{(A_1^{-1} - A_1^{-1} A_2^{-1} A_2^{-1} I - A_1 A_1^{-1} (A_2^{-1} - A_2^{-1} I - A_1 A_1^{-1}))}{(A_2^{-1} (I - A_1^{-1} A_2^{-1} A_2^{-1}) A_2^{-1} (I - A_1 A_1^{-1}))}$$

It is easily checked that a conditional inverse of $A'A$ is $A'A^{-1} = A^*(A^{-1})'$ and further that $(AA^{-1})' = (A^*)'A' = AA^*$, $A^*AA^* = A^*$, $AA^*A = A$. Thus

$$A^{-1} = [I] \begin{bmatrix} c' \end{bmatrix} (A'A)^* A' = [I] \begin{bmatrix} c' \end{bmatrix} A^* = c' (A_2^{-1} (I - A_1 A_1^{-1}) A_2^{-1}) A_2^{-1} (I - A_1 A_1^{-1}).$$

This completes the proof.

Lemma 3.2. Given $g \in W_1$, there exists a vector c in the row space of A_2 such that $g'y$ is the BLUE of $c'x_2$.

Proof: Let $g = (I - A_1 A_1^{-1}) A_2^{-1}$, then $E(g'y) = c' A_2^{-1} (I - A_1 A_1^{-1}) A_2^{-1} y = c' x_2$ (say). Using the same argument as in the proof of the last lemma, the BLUE of $c'x_2$ is

$$(3.9) \quad \hat{c'x_2} = c' A_2^{-1} (I - A_1 A_1^{-1}) A_2^{-1} [A_2^{-1} (I - A_1 A_1^{-1}) A_2^{-1}]^{-1} A_2^{-1} (I - A_1 A_1^{-1}) y.$$

SECTION II

METHOD OF ANALYSIS

The analysis procedure used in the computer program is described here. The method, first used by Padlog for the solution of problems involving plastic flow and creep, is given here in a slightly modified form. The well known formulas for the stiffness of bars and triangular plate elements are first presented. Then the step by step iterative procedure used for the solution of problems in which plastic flow occurs is described.

1. ELEMENT PROPERTIES

The two types of elements to be considered in this analysis (the bar and the triangular plate) are shown in Figure 1. The coordinates of the end points of the bar and the vertices of the triangle are referred to a fixed coordinate system in the plane. The cartesian components of the nodal displacements for each of these elements comprise the element displacement vector X . The ordering of the displacement components is shown in Figure 1. The total element strains designated by the vector ϵ can be expressed in terms of the nodal displacements by an equation of the form

$$\epsilon = BX \quad (1)$$

The stresses are related to the elastic strains $\epsilon^e = \epsilon - \epsilon^p$ by Hooke's law

$$\sigma = C \epsilon^e \quad (2)$$

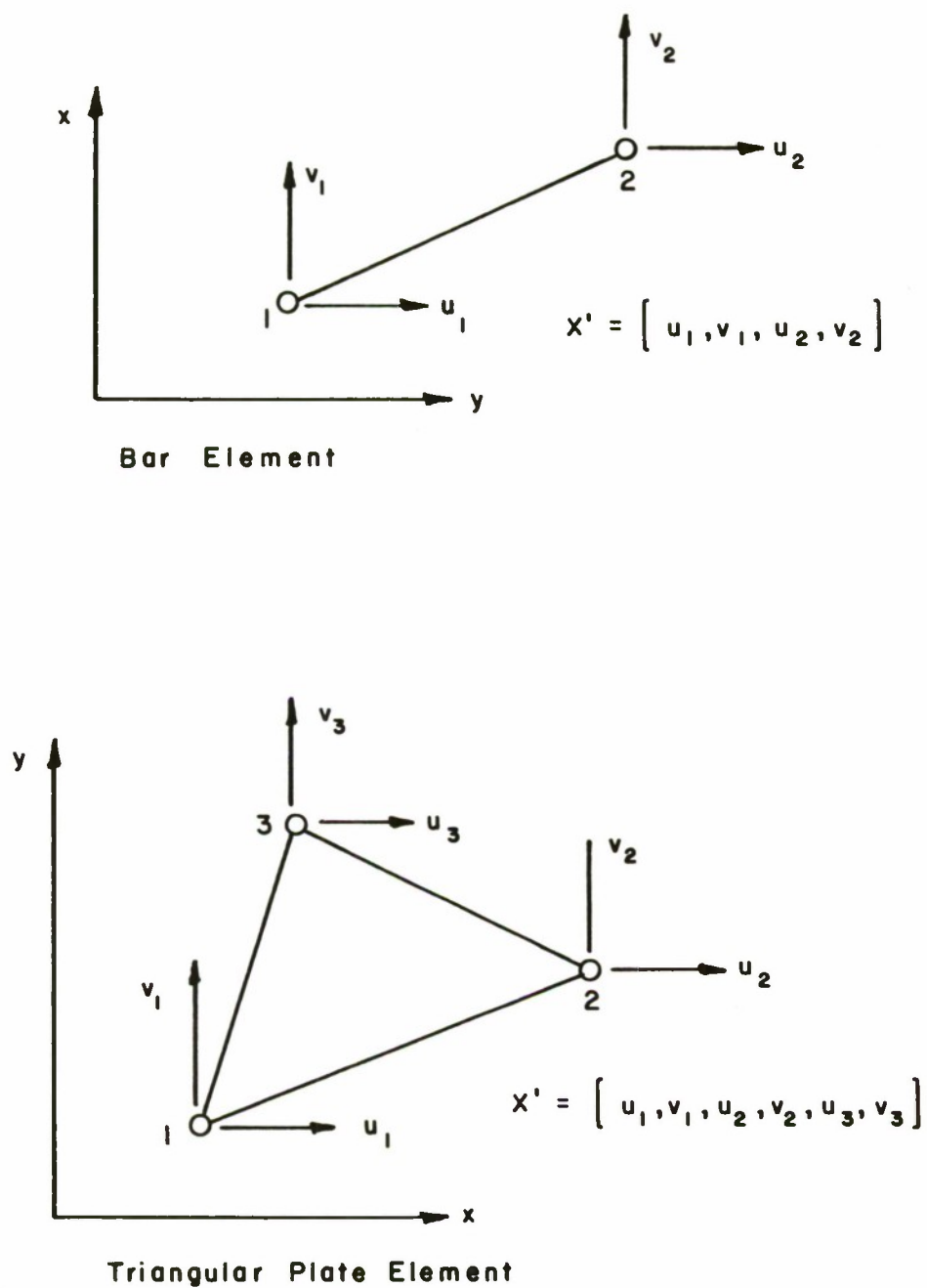


Figure 1. Bar and Plate Elements

The nodal forces F corresponding to given displacements X are found by the principle of virtual work. That is

$$\bar{X}' F = \int_V \bar{\epsilon}' \sigma dV \quad (3)$$

where \bar{X} and $\bar{\epsilon}$ are the virtual displacement vector and the corresponding strain vector, respectively, and the integration is carried out over the volume of the element. Prime superscripts denote transposed matrices. Using Equations 1 and 2 in Equation 3 gives the results

$$\bar{X}' \left[F - \int_V B' C B dV X + \int_V B' C dV \epsilon^P \right] = 0 \quad (4)$$

Or, since the elements of \bar{X} are arbitrary, then

$$F + F^P = k X \quad (5)$$

where

$$k = \int_V B' C B dV \quad (6)$$

is the element stiffness matrix and

$$F^P = D \epsilon^P \quad (7)$$

is the vector of plastic forces in which

$$D = \int_V B' C dV \quad (8)$$

The definitions of the nodal force and displacement vectors and of the matrices C , B , D , and k for bars and triangular plate elements are:

Bar Element

$$X' = [u_1, v_1, u_2, v_2] \quad (9)$$

$$F' = [F_{x1}, F_{y1}, F_{x2}, F_{y2}] \quad (10)$$

$$B = \frac{1}{L^2} [-x_{21}, -y_{21}, x_{21}, y_{21}] \quad (11)$$

in which L is the length of the bar and

$$x_{21} = x_2 - x_1, \quad y_{21} = y_2 - y_1 \text{ etc} \quad (12)$$

$$C = E \text{ -- Young's modulus} \quad (13)$$

$$D' = \frac{AE}{L} \begin{bmatrix} -x_{21} & -y_{21} & x_{21} & y_{21} \end{bmatrix} \quad (14)$$

where A is the cross-sectional area of the bar

$$k = \frac{AE}{L^3} \begin{bmatrix} k_{11} & -k_{11} \\ -k_{11} & k_{11} \end{bmatrix} \quad (15)$$

in which

$$k_{11} = \begin{bmatrix} x_{21}^2 & x_{21} y_{21} \\ x_{21} y_{21} & y_{21}^2 \end{bmatrix} \quad (16)$$

Triangular Plate Element

$$X' = \begin{bmatrix} u_1, v_1, u_2, v_2, u_3, v_3 \end{bmatrix} \quad (17)$$

$$F' = \begin{bmatrix} F_{x1}, F_{y1}, F_{x2}, F_{y2}, F_{x3}, F_{y3} \end{bmatrix} \quad (18)$$

$$\epsilon' = \begin{bmatrix} \epsilon_x, \epsilon_y, \gamma_{xy} \end{bmatrix} \quad (19)$$

$$\sigma' = \begin{bmatrix} \sigma_x, \sigma_y, \tau_{xy} \end{bmatrix} \quad (20)$$

$$B = \frac{1}{h} \begin{bmatrix} -y_{32} & 0 & y_{31} & 0 & -y_{21} & 0 \\ 0 & x_{32} & 0 & -x_{31} & 0 & x_{21} \\ -x_{32} & -y_{32} & -x_{31} & y_{31} & x_{21} & -y_{21} \end{bmatrix} \quad (21)$$

where

$$h = x_{21} y_{31} - x_{31} y_{21} \quad (22)$$

The absolute value of h equals twice the area of the triangular element.

$$C = \frac{E}{1-\nu^2} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & (1-\nu)/2 \end{bmatrix} \quad (23)$$

in which ν = Poisson's ratio

$$D = \frac{E t |h|}{2h(1-\nu^2)} \begin{bmatrix} -y_{32} & -\nu y_{32} & C_1 x_{32} \\ \nu x_{32} & x_{32} & -C_1 y_{32} \\ y_{31} & \nu y_{31} & -C_1 x_{31} \\ \nu x_{31} & -x_{31} & C_1 y_{31} \\ -y_{21} & -\nu y_{21} & C_1 x_{21} \\ \nu x_{21} & x_{21} & -C_1 y_{21} \end{bmatrix} \quad (24)$$

where $C_1 = (1-\nu)/2$ and t is the element thickness.

$$k = \frac{E t}{2|h|(1-\nu^2)} \bar{k} \quad (25)$$

where

$$\bar{k}_{ij} = \bar{k}_{ji} \quad (26)$$

$$\left. \begin{aligned}
\bar{k}_{11} &= y_{32}^2, \bar{k}_{12} = -\nu x_{32} y_{32} - C_1 x_{32} y_{32} \\
\bar{k}_{13} &= -y_{31} y_{32} - C_1 x_{31} x_{32}, \bar{k}_{14} = \nu x_{31} y_{32} + C_1 y_{31} x_{32} \\
\bar{k}_{15} &= y_{21} y_{32} + C_1 x_{21} x_{32}, \bar{k}_{16} = -\nu x_{21} y_{32} - C_1 y_{21} x_{32} \\
\bar{k}_{22} &= x_{32}^2 + C_1 y_{32}^2, \bar{k}_{23} = \nu y_{31} x_{32} + C_1 x_{31} y_{32} \\
\bar{k}_{24} &= -x_{31} x_{32} - C_1 y_{31} y_{32}, \bar{k}_{25} = -\nu y_{21} x_{32} - C_1 x_{21} y_{32} \\
\bar{k}_{26} &= x_{21} x_{32} + C_1 y_{21} y_{32}, \bar{k}_{33} = y_{31}^2 + C_1 x_{31}^2, \\
\bar{k}_{34} &= -\nu x_{31} y_{31} - C_1 y_{31} x_{31}, \bar{k}_{35} = -y_{21} x_{31} - C_1 x_{21} x_{31} \\
\bar{k}_{36} &= \nu x_{21} y_{31} + C_1 y_{21} x_{31}, \bar{k}_{44} = x_{31}^2 + C_1 y_{31}^2, \\
\bar{k}_{45} &= \nu y_{21} x_{31} + C_1 x_{21} y_{31}, \bar{k}_{46} = -x_{21} x_{31} - C_1 y_{21} y_{31} \\
\bar{k}_{55} &= y_{21}^2 + C_1 x_{21}^2, \bar{k}_{56} = -\nu x_{21} y_{21} - C_1 x_{21} y_{21} \\
\bar{k}_{66} &= x_{21}^2 + C_1 y_{21}^2
\end{aligned} \right\} \quad (27)$$

2. ELASTIC-PLASTIC ANALYSIS

In the elastic range of material behavior the equilibrium equations for a structure composed of plate and bar elements of the type considered here can be written

$$F = KX \quad (28)$$

where the force and displacement vectors now have as their components the cartesian components of force and displacement at all the nodes and K is the assembled stiffness matrix for the whole structure. The solution of Equation 28 for the unknown displacement is given symbolically by

$$X = K^{-1} F \quad (29)$$

The displacements known, the element strains can be obtained from Equation 1 and the stresses from Equation 2. However, when the stresses reach the intensity required to cause plastic flow, it becomes necessary to determine the increments of plastic strain caused by the load increment. The material is assumed to obey the Mises yield condition and the associated flow rule. For plane stress the following equations apply

$$\bar{\sigma} = \left(\sigma_x^2 - \sigma_x \sigma_y + \sigma_y^2 + 3 \tau_{xy}^2 \right)^{1/2} = H(\bar{\epsilon}^p) \quad (30)$$

$$\Delta \bar{\epsilon}^p = \frac{2}{\sqrt{3}} \left(\Delta \epsilon_x^p{}^2 + \Delta \epsilon_x^p \Delta \epsilon_y^p + \Delta \epsilon_y^p{}^2 + \frac{1}{4} \gamma_{xy}^p{}^2 \right)^{1/2} \quad (31)$$

$$\left. \begin{aligned} \Delta \epsilon_x^p &= \frac{\Delta \bar{\epsilon}^p}{2 \bar{\sigma}} (2 \sigma_x - \sigma_y) \\ \Delta \epsilon_y^p &= \frac{\Delta \bar{\epsilon}^p}{2 \bar{\sigma}} (2 \sigma_y - \sigma_x) \\ \Delta \gamma_{xy}^p &= 3 \frac{\Delta \bar{\epsilon}^p}{\bar{\sigma}} \tau_{xy} \end{aligned} \right\} \quad (32)$$

where $\bar{\sigma}$ and $\bar{\epsilon}^p$ are the effective stress and the effective plastic strain, respectively, and where $H(\bar{\epsilon}^p)$ is the stress-plastic strain relation for uniaxial stress.

If it is assumed that the response of the structure to the removal of a load increment will be completely elastic then Equation 28 can be modified to account for plastic flow as follows

$$K X = F + F^p \quad (33)$$

where X and F are the displacement and load after the application of the increment and F^p is the vector of plastic forces corresponding to

the plastic strains. The plastic strain increments caused by the increment of load must satisfy Equations 32 and for an element undergoing plastic flow the stresses must satisfy the yield condition (Equation 30).

The following step by step iterative method is used to obtain solutions:

1. An increment is given to the applied loads.
2. New values of displacement are found from Equation 33 using the current values of the plastic forces (these will be zero for the first step).
3. The displacements are used to compute total strains, elastic strains, stresses, and the effective stress.
4. If the new value of the effective stress is greater than the largest previous value, the element is plastic and the effective stress is used to determine a new value of the effective strain.
5. Plastic strain increments computed from Equations 32 are added to the current values of the plastic strain and new values of the plastic forces are calculated.
6. If the increment in effective plastic strain is sufficiently small the iteration is complete and a return to step 1 is made, if not a return is made to step 2 and a new cycle begun.

This procedure is applied to each of the elements and the decision to start a new step (apply a load increment) is based on the largest plastic strain increment found among all the elements.

An important feature of the method is the way in which the effective plastic strain is computed from the new value of the effective stress at each iteration. If the inverse of Equation 30 is used to give $\bar{\epsilon}^P$ as a function of $\bar{\sigma}$ the solution may become unstable. This becomes obvious when one considers the case of the elastic, perfectly plastic material for which the inverse of the function $H(\bar{\epsilon}^P)$ does not exist. To avoid this difficulty the "constant strain" method of Reference 2 is used. In this method the total strain ϵ_t is taken equal to the sum of the value of $\bar{\epsilon}^P$ computed in the previous iteration and $\bar{\sigma}/E$.

The stress-strain law can be written in the form

$$\begin{aligned} \epsilon_t &= \frac{\bar{\sigma}}{E} + \bar{\epsilon}^P \\ \text{or} \\ \epsilon_t &= \frac{H(\bar{\epsilon}^P)}{E} + \bar{\epsilon}^P \end{aligned} \quad (34)$$

The new value of $\bar{\epsilon}^P$ can be found from Equation 34 without difficulty.

The criterion used in step 6 of the iterative procedure given above, to decide whether the plastic strains have been determined with sufficient accuracy, is the size of the ratio of the increment in effective plastic strain to $\bar{\sigma}/E$. This ratio is a measure of the difference between the ordinates to the theoretical stress strain curve and the curve that is actually being used at that step in the calculation.

3. STRESS-STRAIN LAWS

The following three types of stress-strain laws are available for use in the computer program. Each of them is a three parameter law.

Type 1 - Ramberg-Osgood Law

$$\epsilon_t = \frac{\sigma}{E} + \frac{3\sigma_1}{7E} \left(\frac{\sigma}{\sigma_1} \right)^n$$

in which

E — Young's modulus

σ_1 — secant yield stress (stress at which the secant modulus = 0.7E)

n — shape factor

Type 2 - Goldberg-Richard Law

$$\sigma = E \epsilon_t \left[1 + \left| \frac{E \epsilon_t}{\sigma_u} \right|^n \right]^{-1/n}$$

in which

E — Young's modulus

σ_u — maximum stress

n — shape factor

Type 3 - Bilinear Law

$$\begin{aligned} \sigma &= E \epsilon_t, & \text{for } \sigma < \sigma_y \\ \sigma &= \sigma_y + E_1 \left(\epsilon_t - \frac{\sigma_y}{E} \right), & \text{for } \sigma \geq \sigma_y \end{aligned}$$

in which

E — Young's modulus

σ_y — yield stress

E_1 — Slope of the plastic portion of the stress-strain curve

To reduce computing time a linearized form of the Ramberg-Osgood law is used in the program. This law is fitted by a series of straight line segments which match the actual curve at 100 points in the interval $0 \leq \epsilon_p \leq 20 \sigma_y / E$. If values of ϵ_p outside this range are encountered the exact formula is used.

SECTION III

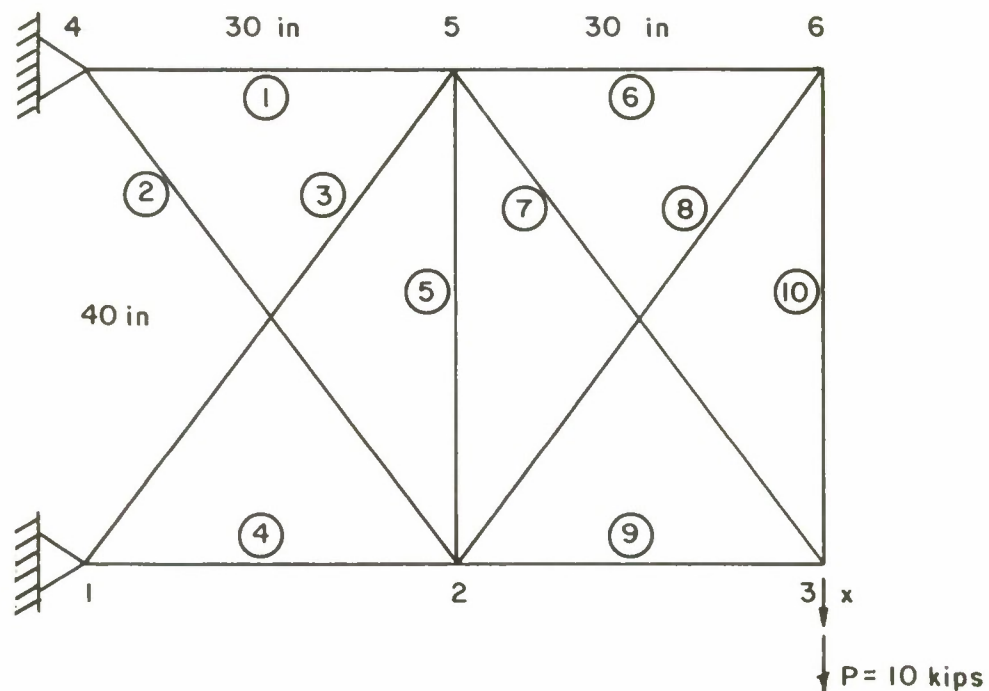
EXAMPLES

1. NONLINEAR TRUSS

To illustrate the use of the program a solution for the member stresses and the tip deflection of the cantilever truss shown in Figure 2 is obtained. A stress-strain relation of the Ramberg-Osgood type is assumed with values of $E = 10.3 \times 10^6$, $\sigma_1 = 40.5 \times 10^3$, and $n = 7$. A solution by another method is given in Reference 6. A comparison of the stresses obtained by the two methods is shown in the table in Figure 2. The tip displacement is shown as a function of the load in Figure 3. The displacements obtained by the two methods agree so closely that both solutions are represented by a single curve.

2. SHEAR LAG SPECIMEN

As a second example, solutions are obtained for the shear lag specimen tested in Reference 3 shown in Figure 4. Two solutions are obtained. In the first solution the same idealization of the structure is used as that used in Reference 3 (see Figure 5). The second solution is found using the idealization shown in Figure 6. Values of the Ramberg-Osgood constants of $E = 10.2 \times 10^6$, $\sigma_1 = 46.6 \times 10^3$, and $n = 10$ were used. These correspond to the values for the RO2 stress-strain curve of Reference 3.



ELEMENT	FORCE, in kips	
	Ref. 6	Present Analysis
1	11.26	11.18
2	6.23	6.19
3	- 6.27	- 6.31
4	-11.24	-11.11
5	- 0.51	- 0.52
6	3.35	3.33
7	6.91	6.95
8	- 5.59	- 5.55
9	- 4.15	- 4.17
10	4.47	4.44

Figure 2. Nonlinear Truss

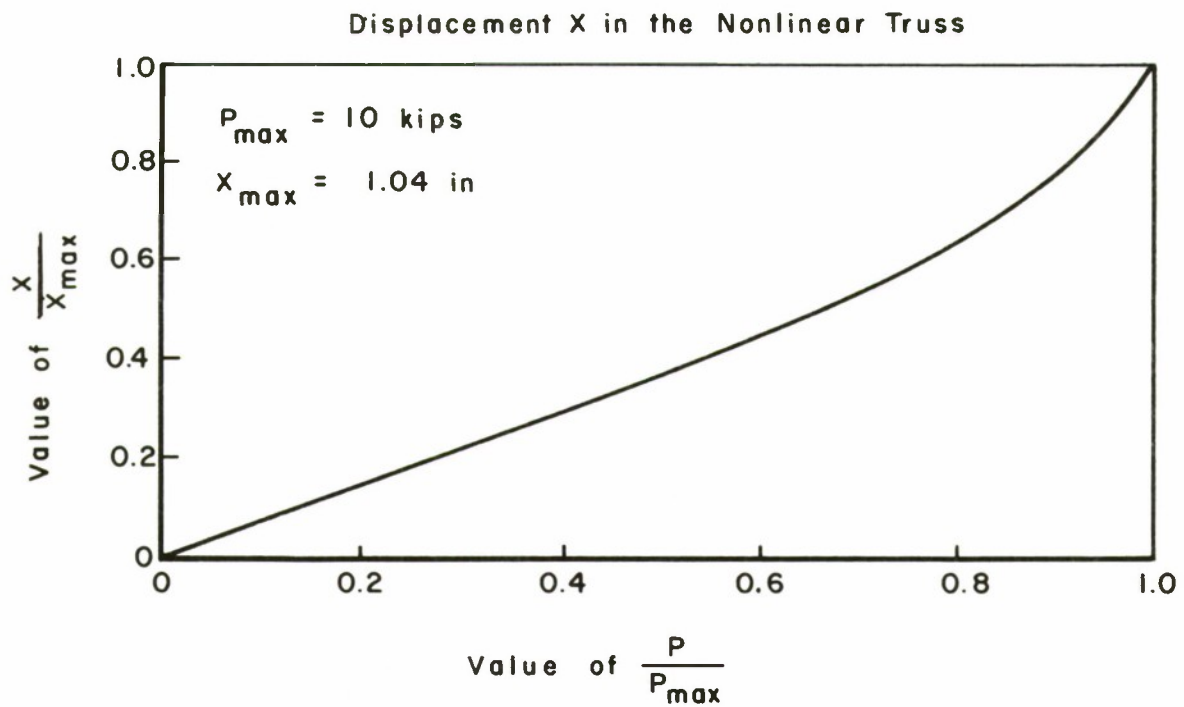


Figure 3. Load Versus Tip Displacement - Nonlinear Truss

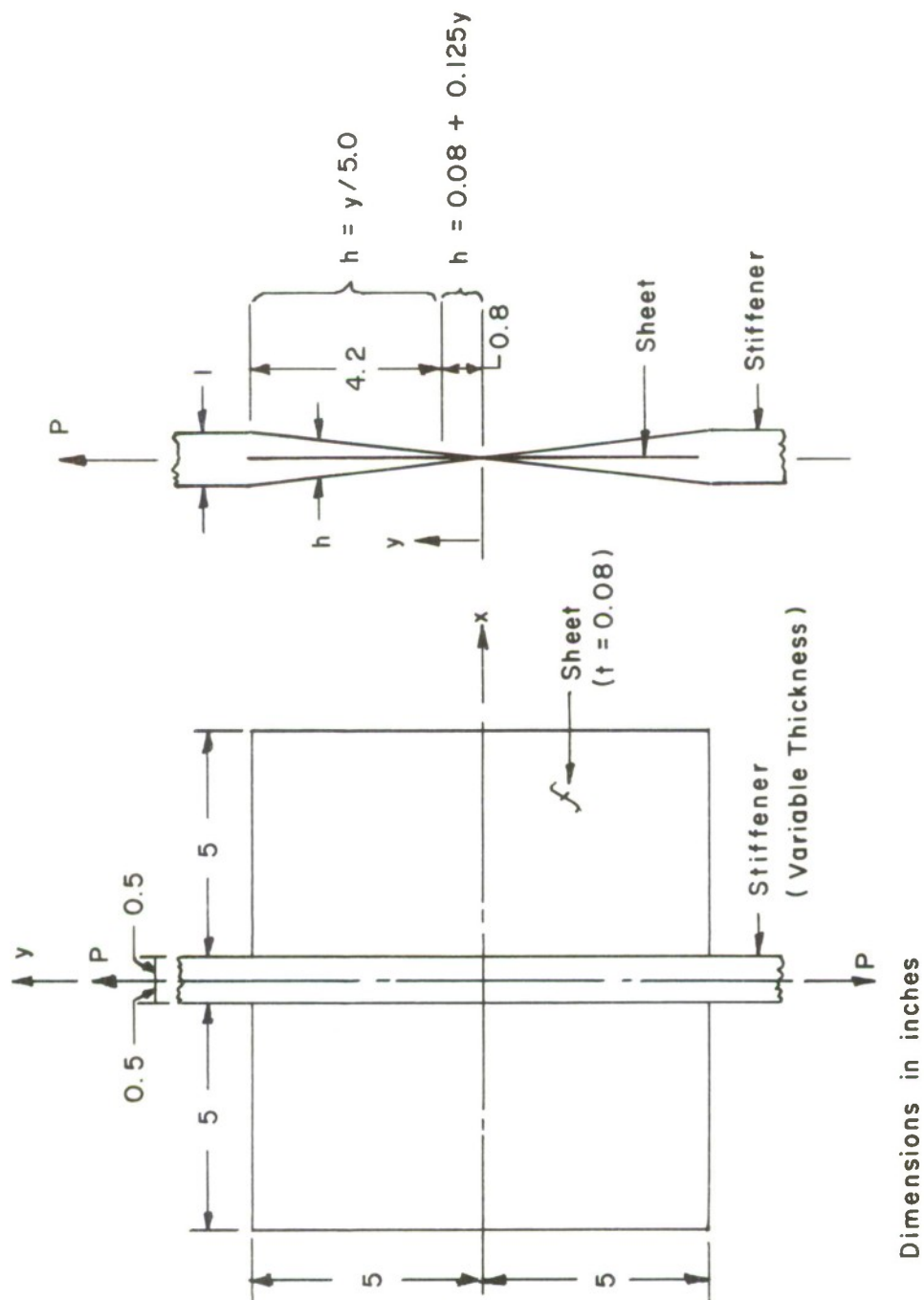


Figure 4. MIT Shear Lag Specimen

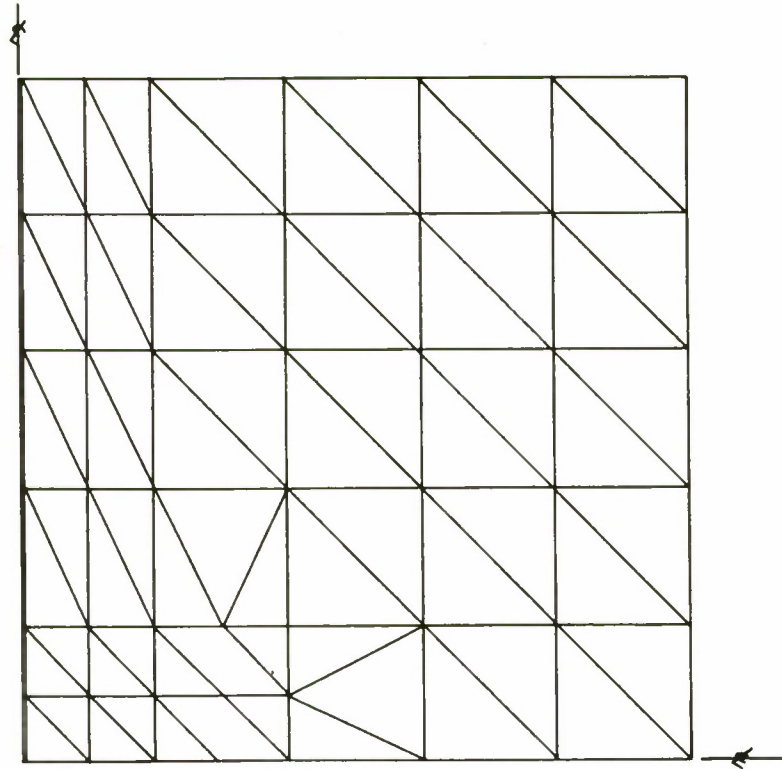


Figure 5. MIT Shear Lag Specimen - Configuration I

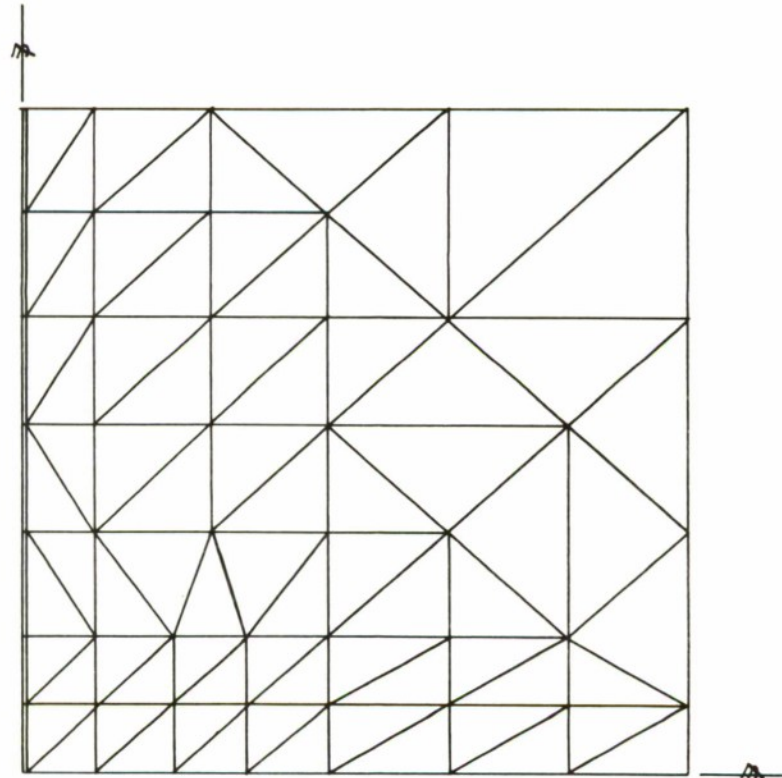


Figure 6. MIT Shear Lag Specimen - Configuration II

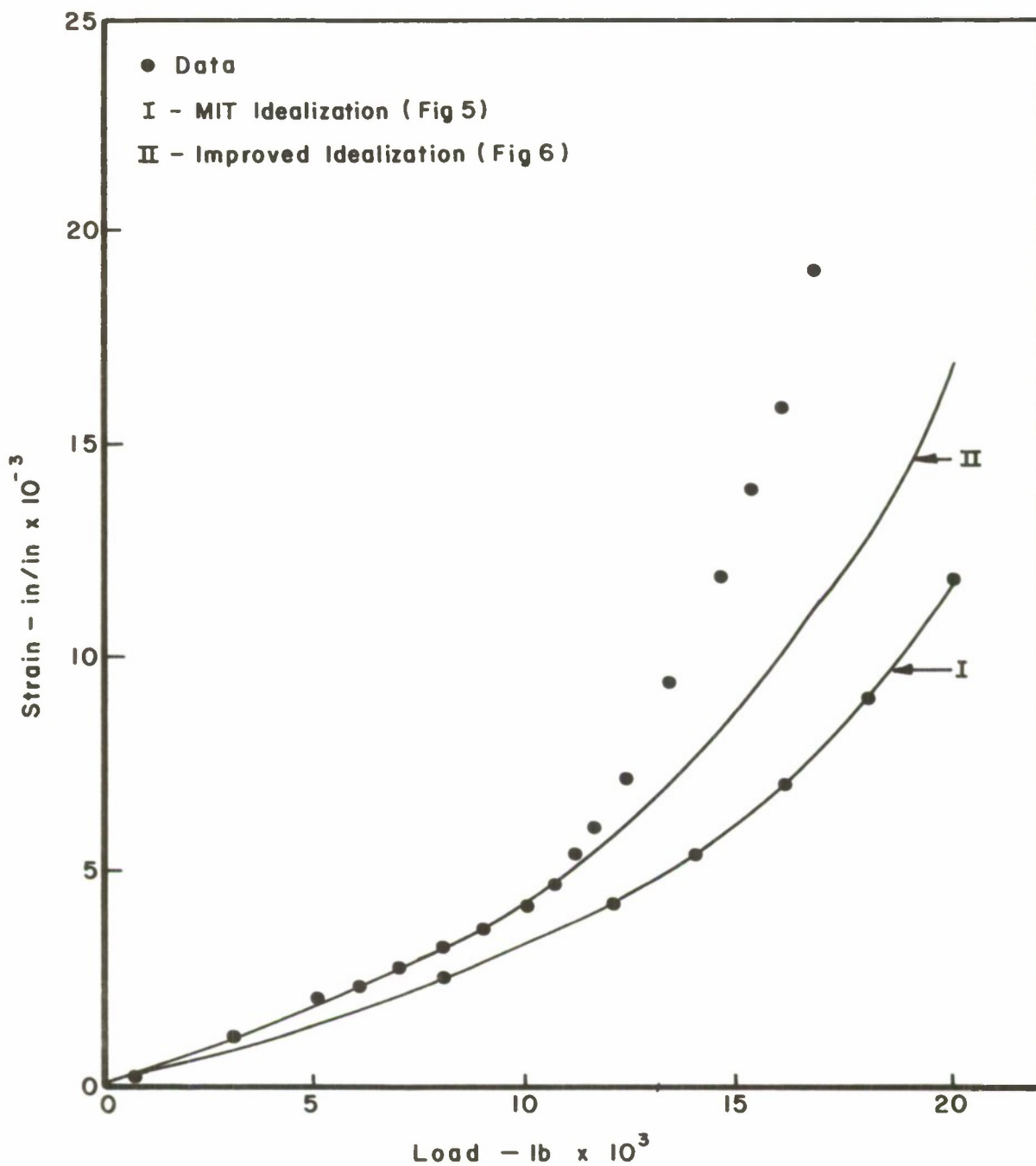


Figure 7. Comparison of Test Results of Shear Lag Specimen with Finite Element Analyses, Axial Strain at Center of Stiffener

A comparison of the results of these solutions with the test data for axial strain at the center of the stiffener is given in Figure 7. While the second solution is in better agreement with the test results than the first, the agreement at large values of the load is poor. A still more refined idealization would probably improve the solution but, as shown in Reference 5, much of the discrepancy is due to the inadequacy of the tensile stress-strain data in the range of large strains.

3. PERFORATED STRIP

Theocaris and Marketos (Reference 8) obtained results for a linear strain-hardening aluminum strip with a ratio of hole diameter to strip width of 1:2. The material properties were

$$\begin{aligned}\text{yield stress} &= 24.3 \text{ kg/mm}^2 \\ \text{plastic modulus} &= 225.0 \text{ kg/mm}^2 \\ \text{Young's modulus} &= 7000.0 \text{ kg/mm}^2\end{aligned}$$

The finite element idealization of the test specimen is shown in Figure 8; 116 nodes and 172 triangular elements were used. A comparison of measured and computed values of the maximum strain in the y direction at the edge of the hole is given in Figure 9. The agreement between theory and experiment is fairly good. The same test is used by Marcal and King (Reference 7) for comparison with the results of their analysis and about the same degree of agreement is obtained.

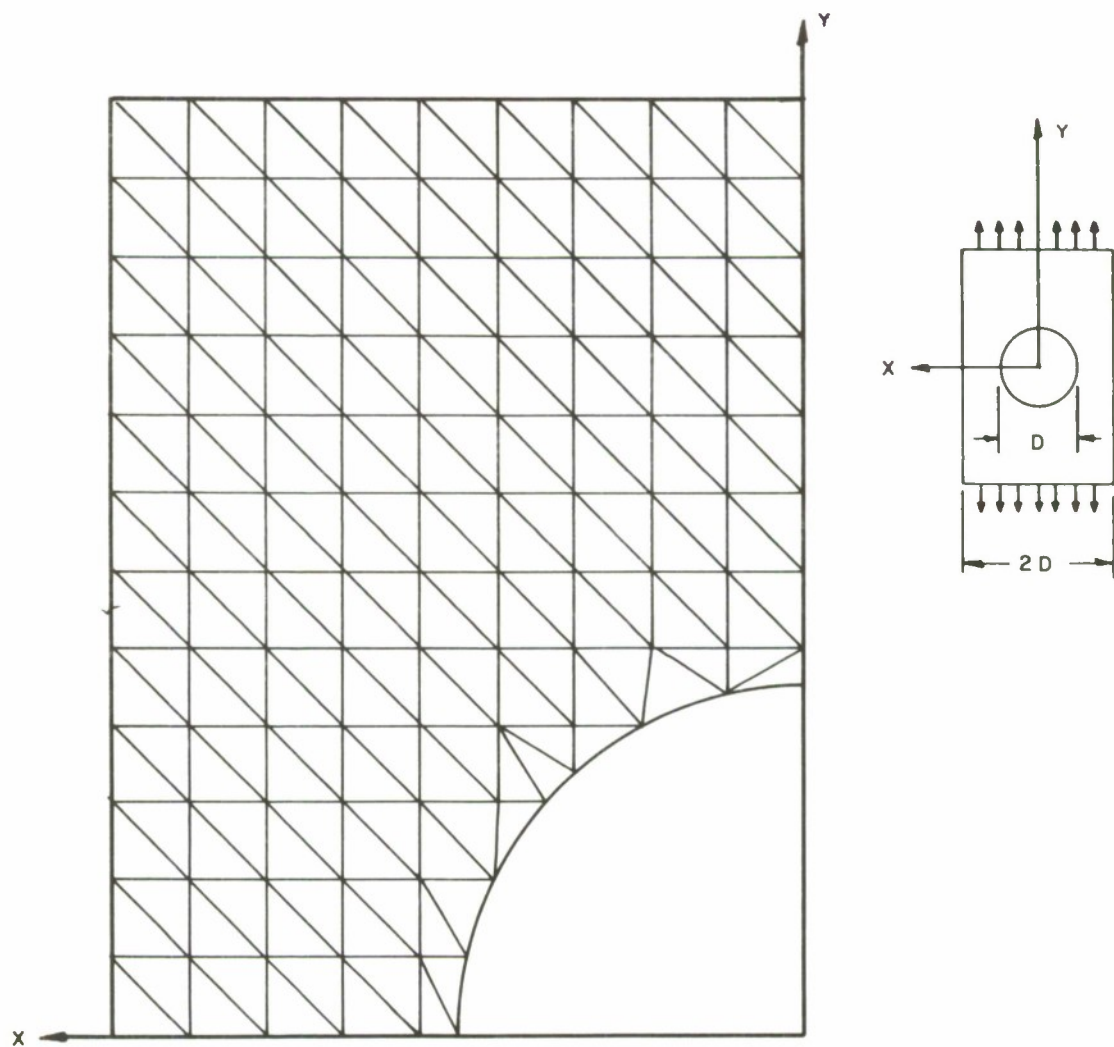


Figure 8. Perforated Strip Finite Element Idealization

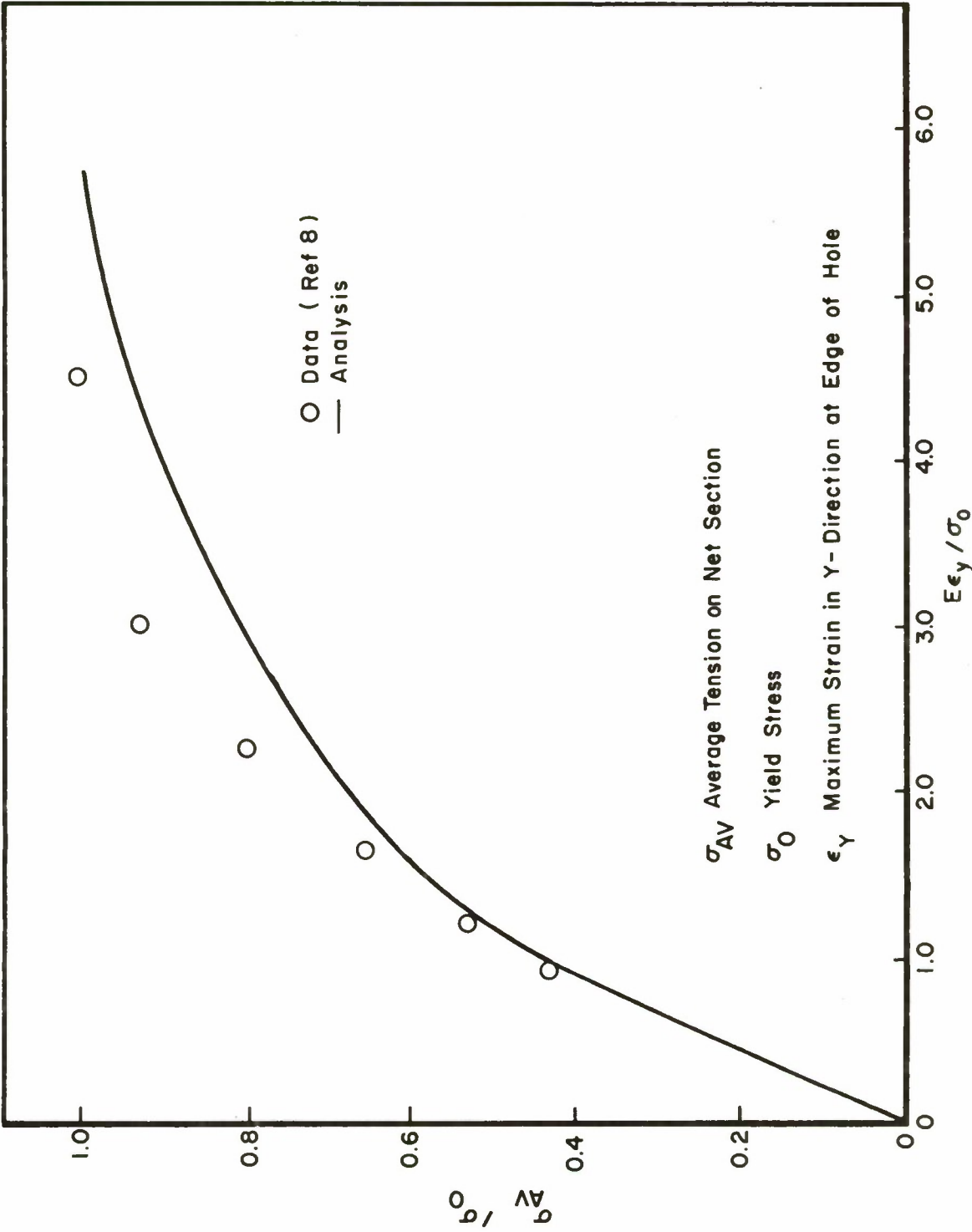


Figure 9. Maximum Values of $E \epsilon_Y / \sigma_0$ for Perforated Strip

SECTION IV

DESCRIPTION OF COMPUTER PROGRAM

1. INTRODUCTION

The FORTRAN IV program for the elastic-plastic analysis of plane structures composed of bar and triangular plate elements is described in this section. The correspondence between the program variables and the stress-strain law parameters for each of the three laws available is given in Table I.

TABLE I

CORRESPONDENCE BETWEEN PROGRAM VARIABLES
AND STRESS-STRAIN LAW PARAMETERS

Stress-Strain Law	Program Variables				
	ILAW	E	EE1	EE2	PRR
Ramberg-Osgood	1	E	σ_1	n	ν
Goldberg-Richard	2	E	σ_u	n	ν
Bilinear	3	E	σ_y	E_1	ν

2. INPUT DATA AND DESCRIPTION OF OUTPUT

The geometry of the structure is determined by specifying the x and y coordinates of each node with respect to a fixed set of coordinate axes and the thickness (cross-sectional area in the case of bars) of the elements. Up to 225 nodes and 400 elements can be handled. The program uses a subroutine for the solution of simultaneous equations in band form written by Professor E. L. Wilson of the University of

California. Great economies in storage requirements and in time required for solution are achieved in this way; however, the bandwidth of the equations defined by the idealization of the structure is limited in size. To meet this limitation on bandwidth the difference between the node numbers on any element must be 9 or less. Instructions for increasing the bandwidth are given in Table II.

The displacement components in the x and y direction can be specified at any node or a node can be required to move along a line with a specified slope. Boundary conditions can be specified at up to 29 nodes.

The x and y components of load can be specified at any node. Distributed loads must be treated as concentrated at the nodes.

The number of equal increments (steps) into which the applied loads and specified displacements are to be divided is specified as input. It is also necessary to specify the number of the increment at which the solution is to start. For example, if a number of increments $NDIV = 20$ is specified and a value of the starting increments $KSTART = 5$ is used, one quarter of the load (displacement) will be applied in the first step, the rest in 15 equal increments. If it is desired to stop the solution at an intermediate step a value of $KSTOP$ may be specified. If the unloading solution is desired the value $IUNLD = 1$ is used.

An error tolerance must be specified as input. After each cycle of iteration the maximum error among all the elements is compared with the specified tolerance. If the tolerance is met the next load

TABLE II
PROGRAM MODIFICATION

The bandwidth is governed by the difference between the node numbers of a given element. The largest such difference J determines the bandwidth in this program by the formula $MBAND = 2 * J + 3$. This number cannot be greater than 22 in the present program. This is a rather small bandwidth, but it allows 225 nodes in a 32K core machine. To change the applicable problem size of the program in terms of the basic problem size parameters the following dimensions have to be changed:

<u>To Change:</u>	<u>Change Dimensions of:</u>
1. Number of materials	EE, EE1, EE2, PRR, and TAB in common statements (presently 10)
2. Number of elements	I1, I2, I3, I4, NTYPE, Z, SEF, SET, EEP, EXPL, EYP, EXYP in common statement, JX in main program, and modify equivalence statements containing JX (presently 400)
3. Number of nodes	B, X, XCORD, Y, ICODE, FP, F in common statements, FE in main program, ICODE in subroutine DCODE, XX in subroutine ELEM (presently 225 or $2 \times 225 = 450$)
4. Bandwidth	B in common statements (presently 22)
5. Number of nodes with boundary conditions	BC in common statements (presently 30)

increment is applied, if not, the iteration is continued. If the tolerance on error is not met when the allowable number of iterations is reached the solution is stopped. A detailed description of the input data format is given in Table III.

TABLE III
INPUT DATA FORMAT*

Card 1	TITLE CARD (72H)		
Col	1-72	Any alphanumeric information	
Card 2	PROPERTIES CARD (1415)		
Col	1- 5	NNODE	- number of nodes (maximum 225)
	6-10	NELEM	- number of elements (maximum 400)
	11-15	ILAW	- 1 Ramberg-Osgood Law
			- 2 Goldberg-Richard Law
			- 3 Bilinear Law
	16-20	IUNLD	- 1 Unloading following loading
			- 0 Loading only
	21-25	MAT	- number of materials used (maximum 10)
	26-30	MAXBND	- maximum bandwidth, MAXBND = 22 for this program
	31-35	NBC	- number of boundary conditions with prescribed displacement. The maximum number is 30 in this program.
Card 3	MATERIAL PROPERTIES CARDS (E15.8, 3F10.5)		
Col	1-15	EE	- modulus of elasticity
	16-25	EE1	- secant yield stress, ultimate stress, yield stress
	26-35	PRR	- Poisson's ratio
	36-45	EE2	- shape parameter, plastic modulus
Card 4	CONTROL CARD (6I5, F10.0)		
Col	1- 5	NDIV	- number of load increments
	6-10	NIT	- maximum number of iterations per step
	11-15	NPRINT	- print output for each NPRINT incre- ment. (e. g., if NPRINT = 3, for increments 3, 6, 9 etc.)
	16-20	KSTART	- number of increments at which solution is to start
	21-25	KSTOP	- number of increments at which solution is to stop
	26-30	NLOAD	- number of nodes at which loads are specified
	31-40	TOL	- error tolerance

TABLE III (CONTD)

Card 5	NODE CARDS (4I5, 5F10.0)		
Col	1- 5	Node number	
	6-10	IBCX	= 1, if displacement in x-direction is specified
	11-15	IBCY	= 1, if displacement in y-direction is specified
	16-20	IBCS	= 1, if slope is specified
	21-30	XCORD	- x coordinate of the node
	31-40	YCORD	- y coordinate of this node
	41-50	BCI	- specified displacement in x-direction
	51-60	BC2	- specified displacement in y-direction
	61-70	BC3	- specified slope at the node
Card 6	ELEMENT CARDS (5I5, F10.0)		
Col	1- 5	Element number	
	6-10	I1	- nodes defining the element
	11-15	I2	- nodes defining the element
	16-20	I3	- nodes defining the element
	21-25	NTYPE	- material type
	26-35	Z	- element thickness or cross-sectional area
Card 7	LOAD CARDS (I5, 2F10.0)		
	1- 5	Node number	
	6-15	x-component of force	
	16-25	y-component of force	

*NOTE: Input data information in Table III is self explanatory. The use of more than one material, however, may need some clarification. The number of materials "MAT" specified in the field of card 2 defines the number of material properties cards. The sequencing of these cards in turn defines "NTYPE" in the element card, for example, if the element uses material specified in the second material properties card, integer 2 is placed in the field corresponding to "NTYPE".

The nodal forces and displacements, the maximum error and the number of the element in which it occurs are printed out at the end of each step (increment). The cartesian components, principal values, and direction of stress and strain are printed out at the user's option by specifying a value of NPRNT as input. For example, a value of NPRNT = 3 will cause the stresses and strains to be printed out for increment numbers divisible by three. The directions of the principal axes of stress are defined by

$$\phi = -\frac{1}{2} \tan^{-1} \frac{2\tau}{\sigma_x - \sigma_y}, -\frac{\pi}{2} < \phi < \frac{\pi}{2}$$

The value of ϕ in degrees is printed out. In the case of strain the principal directions are defined by

$$\phi = -\frac{1}{2} \tan^{-1} \frac{\gamma}{\epsilon_x - \epsilon_y}, -\frac{\pi}{2} < \phi < \frac{\pi}{2}$$

This value is also printed out since in general the principal axes of stress and total strain do not coincide when plastic flow has taken place.

The effective stress and the effective plastic strain are also given as output for each element.

The input data is printed out at the start of the program to aid in problem identification and checking.

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2. J. Padlog, R. D. Huff, and G. F. Holloway, Unelastic Behavior of Structures Subjected to Cyclic, Thermal, and Mechanical Stressing Conditions, WADD-TR-60-271, December 1960.
3. J. H. Percy, W. A. Loden, and D. R. Navaratna, A Study of Matrix Analysis Methods for Inelastic Structures, RDT-TDR-63-4032, October 1963.
4. J. L. Swedlow and W. H. Yang, Stiffness Analysis of Elasto-Plastic Plates, AFRPL-TR-66-5, January 1966.
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7. P. V. Marcal and I. P. King, Elastic-Plastic Analysis of Two-Dimensional Stress Systems by the Finite Element Method, Int. J. Mech. Sci., Vol. 9, pp 143-155, 1967.
8. P. S. Theocaris and E. Marketos, Elastic-Plastic Analysis of Perforated Strips of Strain-Hardening Material, J. Mech Phys. Solids, Vol. 12, pp. 377-390, 1964.

APPENDIX
COMPUTER PROGRAM LISTING

The FORTRAN IV Source Program and three sample data cases are listed. The first case is the nonlinear truss with Ramberg-Osgood representation of one material. This data is associated with Figures 2 and 3. The second case is the same truss problem slightly changed to show the introduction of more than one material. The third case is Configuration I for the MIT test specimen.

The source deck of the computer program described herein can be obtained by contacting AFFDL (FDTR/BERKE), WPAFB-Ohio, 45433. (513-25-53418).

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$IBJOB
$IBFTC VER7
C      ELASTIC PLASTIC FINITE ELEMENT PROGRAM
C      WITH THREE STRESS STRAIN LAW OPTIONS
COMMON/AOD/ EE(10),EE1(10),EE2(10),PRR(10)
COMMON E,CC,G1,E2,PR,EPR,X21,Y21,X31,Y31,X32,Y32,XERR,
1      N2,NELEM,KEL,ILAW,MAT,NBC,
2      B(450,22),BC(30,3),TAB(101,20),IFIX(2),
3      X(450),XCOR0(225),Y(225),ICODE(225),
4      FP(450),F(450),
5      I1(400),I2(400),I3(400),NTYPE(400),Z(400),I4(400),
6      SEF(400),SET(400),EEP(400),EXPL(400),EYP(400),EXYP(400),
7      NNOOE,MBAND
      DIMENSION JX(400,3),FE(450)
      EQUIVALENCE (JX,I1),(JX(401),I2),(JX( 801),I3)
      EQUIVALENCE (IFIX(1),IBCX),(IFIX(2),IBCY)
C
C      **** READ AND PRINT DATA ****
C
10 READ (5,20)
   IONE=1
20 FORMAT(72H BCO INFORMATION
1      )
   WRITE (6,30)
30 FORMAT(1H1)
   WRITE (6,20)
   READ(5,40) NNOOE,NELEM,ILAW,IUNLO,MAT,MAXBND,NBC
40 FORMAT(14I5)
   READ(5,50) (EE(I),EE1(I),PRR(I),EE2(I),I=1,MAT)
50 FORMAT(E15.8,3F10.5)
   READ(5,60)NDIV,NIT,NPRNT,KSTART,KSTOP,NLOAD,TOL
60 FORMAT(6I5,F10.0)
   IF(KSTOP.EQ.0) KSTOP=NDIV
   GO TO(70,90,110),ILAW
70 WRITE(6,80) (I,EE(I),EE1(I),EE2(I),PRR(I),TOL,I=1,MAT)
80 FORMAT(1H014X18HHRAMBERG OSGOOD LAW/
115X,30HMATERIAL----- I3/
215X30HMOOULUS OF ELASTICITY----- E12.4/
315X30HSECANT YIELD STRESS----- E12.4/
415X30HSHAPE PARAMETER----- E12.4/
515X30HPOISSON'S RATIO----- F8.4/
615X30HERROR TOLERANCE----- F8.4)
   GO TO 130
90 WRITE(6,100) (I,EE(I),EE1(I),EE2(I),PRR(I),TOL,I=1,MAT)
100 FORMAT(1H014X20HGOLOBERG RICHARD LAW/
115X,30HMATERIAL----- I3/
215X30HMOOULUS OF ELASTICITY----- E12.4/
315X30HULTIMATE STRESS----- E12.4/
415X30HSHAPE PARAMETER----- E12.4/
515X30HPOISSON'S RATIO----- F8.4/
615X30HERROR TOLERANCE----- F8.4)
   GO TO 130
110 WRITE(6,120) (I,EE(I),EE1(I),EE2(I),PRR(I),TOL,I=1,MAT)
120 FORMAT(1H014X12HBILINEAR LAW/
115X,30HMATERIAL----- I3/
215X30HMOOULUS OF ELASTICITY----- E12.4/
315X30HYIELD STRESS----- E12.4/
415X30HPLASTIC MODULUS----- E12.4/
515X30HPOISSON'S RATIO----- F8.4/
615X30HERROR TOLERANCE----- F8.4)
130 WRITE (6,140)NNOOE,NELEM,NIT,NIV,NIT
140 FORMAT(1H014X30HNO. OF NODES
      NNOOE =14/15X30HNO. OF ELEMVER70060

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1ENTS          NELEM =I4/15X30HNO. OF STEPS          NDIV =I4/15X30VER70061
2HNO. OF ITERATIONS/STEP  NIT =I4)                  VER70062
DO 150 I=1,NBC                                       VER70063
DO 150 J=1,3                                         VER70064
150 BC(I,J)=0                                       VER70065
    IC=1                                             VER70066
    WRITE(6,160)                                     VER70067
160 FORMAT(25H0BOUNDARY CONDITION ARRAY/10H0 NODAL PT15X1HX23X1HY VER70068
    120X7HSLIDING/1H 14X4HCODE7X5HVALUE9X4HCODE7X5HVALUE9X4HCODE VER70069
    27X5HVALUE)                                     VER70070
C                                                     VER70071
C **** NODE COORDINATES AND BOUNDARY CONDITIONS **** VER70072
C                                                     VER70073
DO 200 J=1,NNODE                                     VER70074
READ(5,170) K,I8CX,I8CY,I8CS,XCORD(K),Y(K),BC1,8C2,BC3 VER70075
170 FORMAT(4I5,5F10.0)                             VER70076
    IF(I8CX+I8CY+I8CS.NE.0) WRITE(6,180)K,I8CX,8C1,I8CY,BC2,I8CS,8C3 VER70077
180 FORMAT(17,3X,3(18,1PE17.7))                   VER70078
    ICODE(K)=I8CS+10*I8CY+100*I8CX                 VER70079
    IF(8C1+8C2+8C3.EQ.0.) GO TO 200                 VER70080
    ICODE(K)=ICOD(K)+IC*1000                       VER70081
    BC(IC,1)=BC1                                    VER70082
    BC(IC,2)=BC2                                    VER70083
    BC(IC,3)=BC3                                    VER70084
    IC=IC+1                                          VER70085
    IF(IC.LE.NBC)GO TO 200                          VER70086
    WRITE(6,190)                                     VER70087
190 FORMAT(54H0 MORE THAN 29 NODES HAVE NON ZERO BOUNDARY CONDITIONS) VER70088
    GO TO 620                                       VER70089
200 CONTINUE                                         VER70090
C                                                     VER70091
C **** ELEMENT PROPERTIES ****                     VER70092
C                                                     VER70093
READ(5,210)(K,I1(K),I2(K),I3(K),NTYPE(K),Z(K),J=1,NELEM) VER70094
210 FORMAT(5I5,F10.0)                             VER70095
C                                                     VER70096
C **** LOADS ****                                  VER70097
C                                                     VER70098
N2=2*NNODE                                          VER70099
DO 220 K=1,N2                                       VER70100
220 F(K)=0                                           VER70101
    IF(NLOAD.EQ.0) GO TO 250                       VER70102
    DO 230 K=1,NLOAD                                VER70103
230 READ(5,240)J,F(2*J-1),F(2*J)                 VER70104
240 FORMAT(15,2F10.0)                             VER70105
250 CONTINUE                                         VER70106
    WRITE(6,260)(K,XCORD(K),F(2*K-1),Y(K),F(2*K),ICODE(K),K=1,NNODE) VER70107
260 FORMAT(10H0 NODAL PT8X7HX-COORD8X7HX-FORCE8X7HY-COORD8X7HY-FORCE VER70108
    11X4HCODE/(4X,I3,5X4F15.4,I15))               VER70109
    WRITE(6,270)                                     VER70110
270 FORMAT(1H0///10X,90HELEMENT      NODE 1      NODE 2      NODE 3      ELEMEN VER70111
    1T TYPE      AREA OR THICK.      MATERIAL TYPE,/) VER70112
    DO 300 K=1,NELEM                                VER70113
    IF(I3(K).EQ.0) WRITE(6,280) K,I1(K),I2(K),I3(K),Z(K),NTYPE(K) VER70114
    IF(I3(K).NE.0) WRITE(6,290) K,I1(K),I2(K),I3(K),Z(K),NTYPE(K) VER70115
280 FORMAT( 6X,4I9,11X,3H8AR,E22.5,I14)          VER70116
290 FORMAT( 6X,4I9,11X,5HPLATE,E20.5,I14)         VER70117
300 CONTINUE                                         VER70118
C                                                     VER70119
C INITIALIZATION                                    VER70120
C                                                     VER70121
310 XDIV=NDIV                                         VER70122

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GO TO(320,340,340),ILAW	VER70123
320 CONTINUE	VER70124
DO 330 I=1,MAT	VER70125
E=EE(I)	VER70126
E1=EE1(I)	VER70127
E2=EE2(I)	VER70128
CC=E1/E	VER70129
G1=(7.0*E/3.0)**(1.0/E2)*E1**(1.0-1.0/E2)	VER70130
CALL TABLE(I)	VER70131
330 CONTINUE	VER70132
340 CONTINUE	VER70133
C	VER70134
C **** DETERMINE BAND WIDTH ****	VER70135
C	VER70136
DO 350 K=1,NELEM	VER70137
I4(K)=I3(K)	VER70138
350 IF(I3(K).EQ.0) JX(K,3)=JX(K,1)	VER70139
J=0	VER70140
DO 380 N=1,NELEM	VER70141
DO 380 I=1,3	VER70142
DO 370 L=1,3	VER70143
KK=IABS(JX(N,I)-JX(N,L))	VER70144
IF(KK-J)370,370,360	VER70145
360 J=KK	VER70146
370 CONTINUE	VER70147
380 CONTINUE	VER70148
MBAND=2*J+3	VER70149
IF(MBAND.GT.MAXBND) WRITE(6,390) MBAND	VER70150
IF(MBAND.GT.MAXBND) GO TO 10	VER70151
390 FORMAT(1H010X20H BAND WIDTH TOO LARGE5X6H MBAND=14)	VER70152
DO 400 I=1,NELEM	VER70153
400 I3(I)=I4(I)	VER70154
DO 410 I=1,N2	VER70155
DO 410 J=1,MBAND	VER70156
410 B(I,J)=0.	VER70157
C	VER70158
C CALCULATION OF STIFFNESS MATRIX	VER70159
C	VER70160
CALL STIFF	VER70161
C	VER70162
C **** REDUCE MATRIX ****	VER70163
C	VER70164
CALL SYMSOL(1)	VER70165
C	VER70166
C **** INCREMENT LOADS, ADD PLASTIC FORCES AND SOLVE FOR DISPLACEMENTS **	VER70167
C	VER70168
DO 420 I=1,NELEM	VER70169
SET(I)=0	VER70170
SEF(I)=0	VER70171
EEP(I)=0	VER70172
EXPL(I)=0	VER70173
EYP(I)=0	VER70174
420 EXYP(I)=0	VER70175
K0=KSTART-1	VER70176
KU=K0	VER70177
DO 430 I=1,N2	VER70178
430 FP(I)=0	VER70179
GO TO 490	VER70180
440 WRITE (6,450)KU,(I,FE(2*I-1),FE(2*I),X(2*I-1),X(2*I),I=1,NNODE)	VER70181
450 FORMAT(1H120X38H FORCES AND DISPLACEMENTS FOR INCREMENT,14//10X4HNO	VER70182
1DE5X7HX-FORCE8X7HY-FORCE9X8HX-0ISPL.7X8HY-DISPL./(9X13,2F15.3,5X2E	VER70183
215.4))	VER70184

WRITE (6,460)XERR,KEL,IT	VER70185
460 FORMAT(13HOMAX. ERROR =F8.5,5X14HIN ELEMENT NO.14,5X17HNO. OF ITER	VER70186
1ATIONS14)	VER70187
470 IF(MOD(KU,NPRNT))490,480,490	VER70188
480 CALL OUTPT	VER70189
IF(KU.EQ.KSTOP)GO TO 620	VER70190
GO TO 500	VER70191
490 IF(KO.EQ.KSTOP) CALL OUTPT	VER70192
IF(KO.EQ.KSTOP)GO TO 620	VER70193
500 KO=KU+IONE	VER70194
KU=KU+1	VER70195
IF(KO=NDIV)510,510,620	VER70196
510 XKO=KO	VER70197
DO 520 I=1,N2	VER70198
520 FE(I)=XKO/XDIV*F(I)	VER70199
DO 530 K=1,NELEM	VER70200
530 SEF(K)=SET(K)	VER70201
IT=0	VER70202
540 DO 570 I=1,NNODE	VER70203
IF(ICODE(I).EQ.0) GO TO 570	VER70204
CALL DCODE(ICODE,I,IBCS,IBCX,IBCY,IC,IX,IY,NBC)	VER70205
IF(IBCS.NE.1) GO TO 550	VER70206
ALF=BC(IC,3)	VER70207
FP(IX)=FP(IX)+ALF*FP(IY)	VER70208
FP(IY)=0.	VER70209
550 DO 560 N=1,2	VER70210
IF(IFIX(N).NE.1) GO TO 560	VER70211
IR=IX+N-1	VER70212
FP(IR)=0.	VER70213
560 CONTINUE	VER70214
570 CONTINUE	VER70215
C	VER70216
C **** SOLVE FOR DISPLACEMENTS ****	VER70217
C	VER70218
DO 580 I=1,N2	VER70219
580 X(I)=FE(I)+FP(I)	VER70220
CALL SYMSOL(2)	VER70221
C	VER70222
C CALCULATE TOTAL STRAINS,STRESSES AND PLASTIC	VER70223
C FORCES AND STRAINS FOR EACH ELEMENT	VER70224
C	VER70225
DO 590 I=1,N2	VER70226
590 FP(I)=0	VER70227
XERR=0.0	VER70228
KEL=0	VER70229
CALL STRAIN	VER70230
C	VER70231
C **** PICK LARGEST ERROR AND DETERMINE WHEN TO REITERATE ****	VER70232
C	VER70233
IT=IT+1	VER70234
IF(XERR-TOL)440,440,600	VER70235
600 IF(IT-NIT)540,610,610	VER70236
610 KO=NDIV	VER70237
IUNLD=0	VER70238
GO TO 440	VER70239
620 IF(IUNLD.EQ.0) GO TO 10	VER70240
IUNLD=0	VER70241
IONE=-1	VER70242
KSTOP=0.	VER70243
GO TO 440	VER70244
END	VER70245
\$IBFTC ELM	ELM 0000

SUBROUTINE ELEM(M)	ELM 0001
COMMON/ADD/ EE(10),EE1(10),EE2(10),PRR(10)	ELM 0002
COMMON E,CC,G1,E2,PR,EPR,X21,Y21,X31,Y31,X32,Y32,XERR,	ELM 0003
1 N2,NELEM,KEL,ILAW,MAT,NBC,	ELM 0004
2 B(450,22),BC(30,3),TAB(101,20),IFIX(2),	ELM 0005
3 X(450),XCORD(225),Y(225),ICODE(225),	ELM 0006
4 FP(450),F(450),	ELM 0007
5 I1(400),I2(400),I3(400),NTYPE(400),Z(400),I4(400),	ELM 0008
6 SEF(400),SET(400),EEP(400),EXPL(400),EYP(400),EXYP(400),	ELM 0009
7 NNODE,M&AND	ELM 0010
DIMENSION XX(225)	ELM 0011
EQUIVALENCE (XCORD,XX)	ELM 0012
J1=I1(M)	ELM 0013
J2=I2(M)	ELM 0014
J3=I3(M)	ELM 0015
X21=XX(J2)-XX(J1)	ELM 0016
Y21=Y(J2)-Y(J1)	ELM 0017
IF(J3.EQ.0) GO TO 10	ELM 0018
Y32=Y(J3)-Y(J2)	ELM 0019
Y31=Y(J3)-Y(J1)	ELM 0020
X32=XX(J3)-XX(J2)	ELM 0021
X31=XX(J3)-XX(J1)	ELM 0022
RETURN	ELM 0023
10 Y32=SQRT(X21**2+Y21**2)	ELM 0024
RETURN	ELM 0025
END	ELM 0026
\$IBFTC DCOD	DCOD0000
CDCOD	DCOD0001
SUBROUTINE DCODE(ICODE,I,IBCS,IBCX,IBCY,IC,IX,IY,NBC)	DCOD0002
DIMENSION ICODE(225)	DCOD0003
IBCS=MOD(ICODE(I),10)	DCOD0004
IBCX=MOD(ICODE(I),1000)/100	DCOD0005
IBCY=MOD(ICODE(I),100)/10	DCOD0006
IC=MOD(ICODE(I),100000)/1000	DCOD0007
IX=2*I-1	DCOD0008
IY=IX+1	DCOD0009
IF(IC.EQ.0) IC=NBC	DCOD0010
RETURN	DCOD0011
END	DCOD0012
\$IBFTC PNEW	PNEW0000
C2222 PLASTIC STRAIN DETERMINATION	PNEW0001
SUBROUTINE PNEW1 (EEPK,EET,E,E1,E2)	PNEW0002
J=1	PNEW0003
XU=EET	PNEW0004
XL=0	PNEW0005
10 EEPK=.5*(XL+XU)	PNEW0006
20 Y=EEPK+E1/E*EEPK**(1.0/E2)-EET	PNEW0007
30 YP=1.0+E1/E/E2*EEPK**(1.0/E2-1.0)	PNEW0008
J=J+1	PNEW0009
IF(J-50)40,40,100	PNEW0010
40 IF(Y)50,100,60	PNEW0011
50 XL=EEPK	PNEW0012
GO TO 70	PNEW0013
60 XU=EEPK	PNEW0014
70 XT=EEPK-Y/YP	PNEW0015
IF(XU-XT)10,10,80	PNEW0016
80 IF(XT-XL)10,10,90	PNEW0017
90 EEPK=XT	PNEW0018
DIFF=ABS(Y/YP/EEPK)	PNEW0019
IF(DIFF-.00001)100,100,20	PNEW0020
100 RETURN	PNEW0021
END	PNEW0022

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$IBFTC PLSTR
CTABL      STRAIN-PLASTIC STRAIN TABLE
          SUBROUTINE TABLE(K)
          COMMON/ADD/ EE(10),EE1(10),EE2(10),PRR(10)
          CMMMDN  E,CC,G1,E2,PR,EPR,X21,Y21,X31,Y31,X32,Y32,XERR,
1          N2,NELEM,KEL,ILAW,MAT,NBC,
2          B(450,22),BC(30,3),TAB(101,20),IFIX(2),
3          X(450),XCDRD(225),Y(225),ICODE(225),
4          FP(450),F(450),
5          I1(400),I2(400),I3(400),NTYPE(400),Z(400),I4(400),
6          SEF(400),SET(400),EEP(400),EXPL(400),EYP(400),EXYP(400),
7          NNODE,MBAND
          II2=2*K
          II1=II2-1
          TAB(1,II1)=0.
          TAB(1,II2)=0.
          DO 20 I=1,101
          IF(I-1)20,20,10
10         TAB(I,II1)=FLDAT(I-1)*CC/5.
          EET=TAB(I,II1)
          CALL PNEW1(EEP,K,EET,E,G1,E2)
          TAB(I,II2)=EEP,K
20        CONTINUE
          RETURN
          END
$IBFTC STIF
SUBROUTINE STIFF
COMMON/ADD/ EE(10),EE1(10),EE2(10),PRR(10)
COMMON  E,CC,G1,E2,PR,EPR,X21,Y21,X31,Y31,X32,Y32,XERR,
1      N2,NELEM,KEL,ILAW,MAT,NBC,
2      B(450,22),BC(30,3),TAB(101,20),IFIX(2),
3      X(450),XCDRD(225),Y(225),ICDDE(225),
4      FP(450),F(450),
5      I1(400),I2(400),I3(400),NTYPE(400),Z(400),I4(400),
6      SEF(400),SET(400),EEP(400),EXPL(400),EYP(400),EXYP(400),
7      NNDDE,MBAND
DIMENSION FFP(2),INDDE(3),LNODE(6),DSK(6,6)
EQUIVALENCE (IFIX(1),IBCX),(IFIX(2),IBCY)
EQUIVALENCE(INODE(1),J1),(INODE(2),J2),(INODE(3),J3)
DO 80 M=1,NELEM
J1=I1(M)
J2=I2(M)
J3=I3(M)
CALL ELEM(M)
NTY=NTYPE(M)
E=EE(NTY)
PR=PRR(NTY)
IF(J3.EQ.0) GO TO 30
IF(NTY.LE.MAT) GO TO 10
GO TO 160
C
C **** STIFFNESS MATRIX CALCULATIONS FOR TRIANGULAR ELEMENTS ****
C
10 JMAT=6
AE=E*Z(M)
A123=X21*Y31-X31*Y21
A123=ABS(A123)
ET1=AE/(2.0*A123*(1.0-PR**2))
ET2=AE/(4.0*A123*(1.0+PR))
DSK(1,1)= ET1*Y32**2      +ET2*X32**2
DSK(2,1)=-ET1*PR*Y32*X32  -ET2*Y32*X32
DSK(2,2)= ET1*X32**2      +ET2*Y32**2

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DSK(3,1)=-ET1*Y31*Y32      -ET2*X32*X31      STIF0037
DSK(3,2)= ET1*PR*Y31*X32    +ET2*Y32*X31      STIF0038
DSK(3,3)=ET1*Y31**2          +ET2*X31**2      STIF0039
DSK(4,1)= ET1*PR*Y32*X31    +ET2*Y31*X32      STIF0040
DSK(4,2)=-ET1*X31*X32      -ET2*Y31*Y32      STIF0041
DSK(4,3)=-ET1*PR*Y31*X31    -ET2*Y31*X31      STIF0042
DSK(4,4)= ET1*X31**2        +ET2*Y31**2      STIF0043
DSK(5,1)= ET1*Y21*Y32      +ET2*X32*X21      STIF0044
DSK(5,2)=-ET1*PR*Y21*X32    -ET2*Y32*X21      STIF0045
DSK(5,3)=-ET1*Y31*Y21      -ET2*X31*X21      STIF0046
DSK(5,4)= ET1*PR*Y21*X31    +ET2*Y31*X21      STIF0047
DSK(5,5)= ET1*Y21**2        +ET2*X21**2      STIF0048
DSK(6,1)=-ET1*PR*Y32*X21    -ET2*Y21*X32      STIF0049
DSK(6,2)= ET1*X32*X21      +ET2*Y21*Y32      STIF0050
DSK(6,3)= ET1*PR*Y31*X21    +ET2*Y21*X31      STIF0051
DSK(6,4)=-ET1*X31*X21      -ET2*Y21*Y31      STIF0052
DSK(6,5)=-ET1*PR*Y21*X21    -ET2*Y21*X21      STIF0053
DSK(6,6)= ET1*X21**2        +ET2*Y21**2      STIF0054
DO 20 I=1,JMAT              STIF0055
DO 20 J=1,JMAT              STIF0056
20 DSK(I,J)=DSK(J,I)        STIF0057
GO TO 50                    STIF0058
C                             STIF0059
C **** STIFFNESS MATRIX CALCULATIONS FOR BARS **** STIF0060
C                             STIF0061
30 ET1=Z(M)*E/Y32**3        STIF0062
   FFP(1)=X21                STIF0063
   FFP(2)=Y21                STIF0064
   DO 40 I=1,2               STIF0065
   DO 40 J=1,2               STIF0066
   DSK(I,J)=ET1*FFP(I)*FFP(J) STIF0067
   DSK(I+2,J)=-DSK(I,J)      STIF0068
   DSK(I,J+2)=-DSK(I,J)      STIF0069
40 DSK(I+2,J+2)=DSK(I,J)     STIF0070
   JMAT=4                    STIF0071
C                             STIF0072
C **** INCORPORATION OF ELEMENT MATRICES INTO STIF0073
C                             STIF0074
C                             STIF0075
C                             STIF0076
50 JMAT2=JMAT/2              STIF0077
   K=0                       STIF0078
   DO 60 I=1,JMAT2           STIF0079
   DO 60 J=1,2                STIF0080
   K=K+1                       STIF0081
60 LNODE(K)=2*INODE(I)-2+J    STIF0082
   DO 80 I=1,JMAT             STIF0083
   KI=LNODE(I)                STIF0084
   DO 80 J=1,JMAT             STIF0085
   KJ=LNODE(J)                STIF0086
   IF(KJ-KI)80,70,70          STIF0087
70 K=KJ-KI+1                  STIF0088
   B(KI,K)=B(KI,K)+DSK(I,J)   STIF0089
80 CONTINUE                   STIF0090
C                             STIF0091
C **** DISPLACEMENT BOUNDARY CONDITIONS **** STIF0092
C                             STIF0093
DO 150 I=1,NNODE             STIF0094
  IF(ICODE(I).EQ.C) GO TO 150 STIF0095
  CALL DCODE(ICODE,I,IBCS,IBCX,IBCY,IC,IX,IY,NBC) STIF0096
  IF(IBCS.NE.1) GO TO 110      STIF0097
  ALF=BC(IC,3)                STIF0098
  B(IX,1)=B(IX,1)+ALF*(ALF*(B(IY,1)+1.)+2.*B(IX,2))

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B(IX,2)=-ALF	STIF0099
B(IY,1)=1.	STIF0100
F(IX)=ALF*F(IY)+F(IX)	STIF0101
F(IY)=0	STIF0102
KL=IX-MBAND+2	STIF0103
KU=IX+MBAND-1	STIF0104
IF(KL.LT.1) KL=1	STIF0105
IF(KU.GT.N2) KU=N2	STIF0106
DO 100 K=KL,KU	STIF0107
IF(K.EQ.IX.OR.K.EQ.IY) GO TO 100	STIF0108
IF(K.GT.IY) GO TO 90	STIF0109
L=IX-K+1	STIF0110
B(K,L)=B(K,L)+ALF*B(K,L+1)	STIF0111
B(K,L+1)=0.	STIF0112
GO TO 100	STIF0113
90 L=K-IX+1	STIF0114
B(IX,L)=B(IX,L)+ALF*B(IY,L-1)	STIF0115
B(IY,L-1)=0.	STIF0116
100 CONTINUE	STIF0117
110 DO 140 N=1,2	STIF0118
IF(IFIX(N).NE.1) GO TO 140	STIF0119
IR=IX+N-1	STIF0120
ML=IR-MBAND+1	STIF0121
MU=IR+MBAND-1	STIF0122
IF(ML.LT.1) ML=1	STIF0123
IF(MU.GT.N2) MU=N2	STIF0124
DO 130 M=ML,MU	STIF0125
L=IR-M+1	STIF0126
IF(L.LE.1) GO TO 120	STIF0127
F(M)=F(M)-B(M,L)*BC(IC,N)	STIF0128
B(M,L)=0.	STIF0129
GO TO 130	STIF0130
120 L=M-IR+1	STIF0131
F(M)=F(M)-B(IR,L)*BC(IC,N)	STIF0132
B(IR,L)=0.	STIF0133
130 CONTINUE	STIF0134
B(IR,1)=1.	STIF0135
F(IR)=BC(IC,N)	STIF0136
140 CONTINUE	STIF0137
150 CONTINUE	STIF0138
RETURN	STIF0139
160 WRITE(6,170) M	STIF0140
170 FORMAT(1H010X32HINVALID ELEMENT CODE ELEMENT NO.14)	STIF0141
STOP	STIF0142
END	STIF0143
C	STIF0144
\$IBFTC SYMSL	SYMS0000
SUBROUTINE SYMSOL(KKK)	SYMS0001
COMMON/ADD/ EE(10),EE1(10),EE2(10),PRR(10)	SYMS0002
COMMON E,CC,G1,E2,PR,EPR,X21,Y21,X31,Y31,X32,Y32,XERR,	SYMS0003
1 N2,NELEM,KEL,ILAW,MAT,NBC,	SYMS0004
2 B(450,22),PC(30,3),TAB(101,20),IFIX(2),	SYMS0005
3 X(450),XCORD(225),Y(225),ICODE(225),	SYMS0006
4 FP(450),F(450),	SYMS0007
5 I1(400),I2(400),I3(400),NTYPE(400),Z(400),I4(400),	SYMS0008
6 SEF(400),SET(400),EEP(400),EXPL(400),EYP(400),EXYP(400),	SYMS0009
7 NNODE,MBAND	SYMS0010
C	SYMS0011
NN=N2	SYMS0012
MM=MBAND	SYMS0013
GO TO (10,60),KKK	SYMS0014

C		SYMS0015
C	REDUCE MATRIX	SYMS0016
C		SYMS0017
	10 DO 50 N=1,NN	SYMS0018
	DO 40 L=2,MM	SYMS0019
	C=B(N,L)/B(N,1)	SYMS0020
	I=N+L-1	SYMS0021
	IF(NN-I) 40,20,20	SYMS0022
	20 J=0	SYMS0023
	DO 30 K=L,MM	SYMS0024
	J=J+1	SYMS0025
	30 B(I,J)=B(I,J)-C*B(N,K)	SYMS0026
	40 B(N,L)=C	SYMS0027
	50 CONTINUE	SYMS0028
	GO TO 130	SYMS0029
C		SYMS0030
C	REDUCE VECTOR	SYMS0031
C		SYMS0032
	60 DO 80 N=1,NN	SYMS0033
	DO 70 L=2,MM	SYMS0034
	I=N+L-1	SYMS0035
	IF(NN-I) 80,70,70	SYMS0036
	70 X(I)=X(I)-B(N,L)*X(N)	SYMS0037
	80 X(N)=X(N)/B(N,1)	SYMS0038
C		SYMS0039
C	BACK SUBSTITUTION	SYMS0040
C		SYMS0041
	N=NN	SYMS0042
	90 N=N-1	SYMS0043
	IF(N) 100,130,100	SYMS0044
	100 DO 120 K=2,MM	SYMS0045
	L=N+K-1	SYMS0046
	IF(NN-L) 120,110,110	SYMS0047
	110 X(N)=X(N)-B(N,K)*X(L)	SYMS0048
	120 CONTINUE	SYMS0049
	GO TO 90	SYMS0050
C		SYMS0051
	130 RETURN	SYMS0052
C		SYMS0053
	END	SYMS0054
\$IBFTC	STRN	STRN0000
	SUBROUTINE STRAIN	STRN0001
	COMMON/ADD/ EE(10),EE1(10),EE2(10),PRR(10)	STRN0002
	COMMON E,CC,G1,E2,PR,EPR,X21,Y21,X31,Y31,X32,Y32,XERR,	STRN0003
	1 N2,NELEM,KEL,ILAW,MAT,NBC,	STRN0004
	2 B(450,22),BC(30,3),TAB(101,20),IFIX(2),	STRN0005
	3 X(450),XCORD(225),Y(225),ICODE(225),	STRN0006
	4 FP(450),F(450),	STRN0007
	5 I1(400),I2(400),I3(400),NTYPE(400),Z(400),I4(400),	STRN0008
	6 SEF(400),SET(400),EEP(400),EXPL(400),EYP(400),EXYP(400),	STRN0009
	7 NNODE,MBAND	STRN0010
	DO 130 K=1,NELEM	STRN0011
	J1=2*I1(K)-1	STRN0012
	J2=2*I1(K)	STRN0013
	J3=2*I2(K)-1	STRN0014
	J4=2*I2(K)	STRN0015
	J5=2*I3(K)-1	STRN0016
	J6=2*I3(K)	STRN0017
	CALL ELEM(K)	STRN0018
	NTY=NTYPE(K)	STRN0019
	E=EE(NTY)	STRN0020
	E1=EE1(NTY)	STRN0021


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E2=EE2( NTY)
CC=E1/E
PR=PRR( NTY)
IF(ILAW.EQ.1) G1=(7.*E/3.)*(1./E2)*E1*(1.-1./E2)
IF(ILAW.GT.1) G1=E1
EPR=E/(1.0-PR*PR)
IF(I3(K).EQ.0) GO TO 60
C
C **** TRIANGULAR ELEMENT CALCULATIONS ****
C
10 A123=X21*Y31-X31*Y21
SN=A123/ABS(A123)
EXT=(-Y32*X(J1)+Y31*X(J3)-Y21*X(J5))/A123
EYT=(X32*X(J2)-X31*X(J4)+X21*X(J6))/A123
EXYT=(X32*X(J1)-Y32*X(J2)-X31*X(J3)+Y31*X(J4)
1 +X21*X(J5)-Y21*X(J6))/A123
EXE=EXT-EXPL(K)
EYE=EYT-EYP(K)
EXYE=EXYT-EXYP(K)
SX=EPR*(EXE+PR*EYE)
SY=EPR*(EYE+PR*EXE)
SXY=E/(1.0+PR)*EXYE/2.0
SE=SQRT(SX**2-SX*SY+SY**2+3.0*SXY**2)
CRIT=ABS(SE)-SEF(K)
IF(CRIT)40,40,20
20 EET=SE/E+EEP(K)
CALL STRSTN(EET,EEPK,SETK,NTY)
DEEP=EEPK-EEP(K)
30 EEP(K)=SEPK
SETK=SETK
EXPL(K)=DEEP/SE*(SX-SY/2.0)+EXPL(K)
EYP(K)=DEEP/SE*(SY-SX/2.0)+EYP(K)
EXYP(K)=3.0*DEEP/SE*SXY+EXYP(K)
ERR=E*DEEP/SE
ERR=ABS(ERR)
GO TO 50
40 ERR=G.0
50 CONTINUE
Q1=E*Z(K)/(1.0-PR**2)/2.0*SN
Q2=E*Z(K)/(1.0+PR)/4.0*SN
EXPT=EXPL(K)
EYPT=EYP(K)
EXYPT=EXYP(K)
FP(J1)=-Q1*Y32*EXPT-Q1*Y32*PR*EYPT+Q2*X32*EXYPT+FP(J1)
FP(J2)= Q1*X32*PR*EXPT+Q1*X32*EYPT-Q2*Y32*EXYPT+FP(J2)
FP(J3)= Q1*Y31*EXPT+Q1*Y31*PR*EYPT-Q2*X31*EXYPT+FP(J3)
FP(J4)=-Q1*X31*PR*EXPT-Q1*X31*EYPT+Q2*Y31*EXYPT+FP(J4)
FP(J5)=-Q1*Y21*EXPT-Q1*Y21*PR*EYPT+Q2*X21*EXYPT+FP(J5)
FP(J6)= Q1*X21*PR*EXPT+Q1*X21*EYPT-Q2*Y21*EXYPT+FP(J6)
GO TO 110
C
C **** BAR CALCULATIONS ****
C
60 EET=(X21*(X(J3)-X(J1))+Y21*(X(J4)-X(J2)))/Y32**2
STRN=ABS(EET-EXPL(K))
SIGN=(EET-EXPL(K))/STRN
SE=E*STRN
CRIT=SE-SEF(K)
EET=STRN+EEP(K)
IF(CRIT)90,90,70
70 CONTINUE
CALL STRSTN(EET,EEPK,SETK,NTY)

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DEEP=EEP(K)-EEP(K)
80 EEP(K)=EEP(K)
   SET(K)=SET(K)
   EXPL(K)=EXPL(K)+SIGN*DEEP
   ERR=E*DEEP/SE
   ERR=ABS(ERR)
   GO TO 100
90 ERR=0.0
100 CONTINUE
   EXPT=EXPL(K)
   Q1=E*Z(K)/Y32
   FP(J1)=FP(J1)-Q1*X21*EXPT
   FP(J2)=FP(J2)-Q1*Y21*EXPT
   FP(J3)=FP(J3)+Q1*X21*EXPT
   FP(J4)=FP(J4)+Q1*Y21*EXPT
110 IF(ERR-XERR)130,130,120
120 XERR=ERR
   KEL=K
130 CONTINUE
   RETURN
END
$IBFTC STRST
SUBROUTINE STRSTN(EET,EEP(K),SET(K),NTY)
COMMON/ADD/ EE(10),EE1(10),EE2(10),PRR(10)
COMMON E,CC,G1,E2,PR,EPR,X21,Y21,X31,Y31,X32,Y32,XERR,
1 N2,NELEM,KEL,ILAW,MAT,NBC,
2 B(450,22),BC(30,3),TAB(101,20),IFIX(2),
3 X(450),XCORD(225),Y(225),ICODE(225),
4 FP(450),F(450),
5 I1(400),I2(400),I3(400),NTYPE(400),Z(400),I4(400),
6 SEF(400),SET(400),EEP(400),EXPL(400),EYP(400),EXYP(400),
7 NNODE,MBAND
GO TO(10,50,60),ILAW
10 J=5.0*EET/CC+1.0
   NT2=2*NTY
   NT1=NT2-1
   IF(J-101)20,30,30
20 EEPK=TAB(J,NT2)+(TAB(J+1,NT2)-TAB(J,NT2))*(EET-TAB(J,NT1))/
1 (TAB(J+1,NT1)-TAB(J,NT1))
   GO TO 40
30 CALL PNEW1(EEP(K),EET,E,G1,E2)
40 SETK=G1*EEP(K)**(1.0/E2)
   RETURN
50 SETK=E*EET/(1.+(ABS(E*EET/G1))**E2)**(1./E2)
   EEPK=EET-SETK/E
   RETURN
60 EC=G1/E
   IF(EET-EC)70,70,80
70 EEPK=0.
   SETK=E*EET
   RETURN
80 EEPK=EET-EC
   SETK=G1+E2*EEP(K)
   RETURN
END
$IBFTC OUTPUT
C12345 OUTPUT SUBROUTINE
SUBROUTINE OUTPUT
COMMON/ADD/ EE(10),EE1(10),EE2(10),PRR(10)
COMMON E,CC,G1,E2,PR,EPR,X21,Y21,X31,Y31,X32,Y32,XERR,
1 N2,NELEM,KEL,ILAW,MAT,NBC,
2 B(450,22),BC(30,3),TAB(101,20),IFIX(2),

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STRN0084
STRN0085
STRN0086
STRN0087
STRN0088
STRN0089
STRN0090
STRN0091
STRN0092
STRN0093
STRN0094
STRN0095
STRN0096
STRN0097
STRN0098
STRN0099
STRN0100
STRN0101
STRN0102
STRN0103
STRN0104
STRS0000
STRS0001
STRS0002
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STRS0023
STRS0024
STRS0025
STRS0026
STRS0027
STRS0028
STRS0029
STRS0030
STRS0031
STRS0032
STRS0033
OTPU0000
OTPU0001
OTPU0002
OTPU0003
OTPU0004
OTPU0005
OTPU0006

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3      X(450),XCORD(225),Y(225),ICODE(225),      OTPU0007
4      FP(450),F(450),      OTPU0008
5      I1(400),I2(400),I3(400),NTYPE(400),Z(400),I4(400),      OTPU0009
6      SEF(400),SET(400),EEP(400),EXPL(400),EYP(400),EXYP(400),      OTPU0010
7      NNODE,MBAND      OTPU0011
      L=0      OTPU0012
      DO 70 K=1,NELEM      OTPU0013
C      OTPU0014
C      **** TRIANGULAR ELEMENT CALCULATIONS ****      OTPU0015
C      OTPU0016
      NTY=NTYPE(K)      OTPU0017
      E=EE(NTY)      OTPU0018
      PR= PRR(NTY)      OTPU0019
      EPR=E/(1.-PR*PR)      OTPU0020
      IF(I3(K).EQ.0) GO TO 70      OTPU0021
10  CALL ELEM(K)      OTPU0022
      A123=X21*Y31-X31*Y21      OTPU0023
      J1=2*I1(K)-1      OTPU0024
      J2=2*I1(K)      OTPU0025
      J3=2*I2(K)-1      OTPU0026
      J4=2*I2(K)      OTPU0027
      J5=2*I3(K)-1      OTPU0028
      J6=2*I3(K)      OTPU0029
      EXT=(-Y32*X(J1)+Y31*X(J3)-Y21*X(J5))/A123      OTPU0030
      EYT=( X32*X(J2)-X31*X(J4)+X21*X(J6))/A123      OTPU0031
      EXYT=(X32*X(J1)-Y32*X(J2)-X31*X(J3)+Y31*X(J4)      OTPU0032
1      +X21*X(J5)-Y21*X(J6))/A123      OTPU0033
      EXE=EXT-EXPL(K)      OTPU0034
      EYE=EYT-EYP(K)      OTPU0035
      EXYE=EXYT-EXYP(K)      OTPU0036
      SX=EPR*(EXE+PR*EYE)      OTPU0037
      SY=EPR*(EYE+PR*EXE)      OTPU0038
      SXY=E/(1.0+PR)*EXYE/2.0      OTPU0039
      PE2=SQRT((.5*(SX-SY))**2+SXY**2)      OTPU0040
      PHI=.5*ATAN2((-2.0*SXY),(SX-SY))*57.29578      OTPU0041
      PH2=.5*ATAN2((-EXYT),(EXT-EYT))*57.29578      OTPU0042
      PS1=.5*(SX+SY)      OTPU0043
      SIGE1=PS1+PE2      OTPU0044
      SIGE2=PS1-PE2      OTPU0045
      PST1=.5*(EXT+EYT)      OTPU0046
      PET2=SQRT((.5*(EXT-EYT))**2+EXYT**2/4.0)      OTPU0047
      STRE1=PST1+PET2      OTPU0048
      STRE2=PST1-PET2      OTPU0049
      PET2=2.0*PET2      OTPU0050
      N1=I1(K)      OTPU0051
      N2=I2(K)      OTPU0052
      N3=I3(K)      OTPU0053
      XC=XCORD(N1)+(X21+X31)/3.0      OTPU0054
      YC=Y(N1)+(Y21+Y31)/3.0      OTPU0055
      EPE=2.*SQRT((EXPL(K)**2+EXPL(K)*EYP(K)+EYP(K)**2+EXYP(K)**2/4.)/3.0)      OTPU0056
1)      SE=SQRT(SX**2-SX*SY+SY**2+3.*SXY**2)      OTPU0057
      L=L+1      OTPU0058
      IF(MOD(L-1,14))30,20,30      OTPU0060
20  WRITE (6,40)      OTPU0061
30  WRITE(6,50)K,XC,YC,SX,SY,SXY,SIGE1,SIGE2,PE2,PHI,PH2,EXT,EYT,      EXOTPU0062
      1YT,STRE1,STRE2,PET2      OTPU0063
40  FORMAT('H1EL. NO./5X11HCOORDINATES28X33HS T R E S S E S / S T R A      OTPU0064
      11 N S /8H      PHI7X1HX8X1HY6X9H      TAU-XX6X9H      TAU-YY6X9H      TAU-XOTPU0065
      2Y8X7HMAXIMUM8X7HMINIMUM6X9HMAX SHEAR )      OTPU0066
50  FORMAT(1H0I7,0PF8.3,F9.3,1P6E15.4/1H 0PF7.2,F8.2,9X1P6E15.4)      OTPU0067
      WRITE(6,60) SE,EPE      OTPU0068.

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60 FORMAT(1H 23H***** EFFECTIVE STRESS=E12.5,23H***** EFFECTIVE STRAIOTPU0069
1N=E12.5) OTPU0070
70 CONTINUE OTPU0071
C OTPU0072
C ***** BAR CALCULATIONS ***** OTPU0073
C OTPU0074
J=0 OTPU0075
DO 120 K=1,NELEM OTPU0076
NTY=NTYPE(K) OTPU0077
E=EE( NTY) OTPU0078
PR=PRR( NTY) OTPU0079
J1=2*I1(K)-1 OTPU0080
J2=2*I1(K) OTPU0081
J3=2*I2(K)-1 OTPU0082
J4=2*I2(K) OTPU0083
IF(I3(K).NE.0) GO TO 120 OTPU0084
80 CALL ELEM(K) OTPU0085
EET=(X21*(X(J3)-X(J1))+Y21*(X(J4)-X(J2)))/Y32**2 OTPU0086
SE=E*(EET-EXPL(K)) OTPU0087
SEMZK=SE*Z(K) OTPU0088
K1=I1(K) OTPU0089
K2=I2(K) OTPU0090
IF(J)100,90,100 OTPU0091
90 WRITE (6,130) OTPU0092
J=1 OTPU0093
100 CONTINUE OTPU0094
110 WRITE(6,140)K,K1,K2,SE,EET,SEMZK OTPU0095
120 CONTINUE OTPU0096
130 FORMAT(9H0BAR NO. 6X10HNODE NOS. 8X7HSTRESS 8X7HSTRAIN,4X, OTPU0097
113HMEMBER FORCES) OTPU0098
140 FORMAT(1H0 3I8,2E15.5,2X,E15.5) OTPU0099
RETURN OTPU0100
END OTPU0101

```

\$DATA

NONLINEAR TRUSS PROC. ASCE DEC. 1965 ONE MATERIAL RAMBERG OSGOOD LAW

```

6 10 1 1 1 22 30
0.10000000E+05 40.50000 0.3 7.0
10 20 1 1 10 1 .01
1 1 1
2 30.
3 60.
4 1 1 40.
5 30. 40.
6 60. 40.
1 4 5 0 1 .25
2 4 2 0 1 .20
3 1 5 0 1 .20
4 1 2 0 1 .25
5 2 5 0 1 .20
6 5 6 0 1 .25
7 5 3 0 1 .20
8 2 6 0 1 .20
9 2 3 0 1 .25
10 3 6 0 1 .20
3 -10.

```

NONLINEAR TRUSS PROC. ASCE DEC. 1965 TWO MATERIALS RAMBERG OSGOOD LAW

```

6 10 1 0 2 22 30
0.10000000E+05 65.00000 0.3 8.0
0.10000000E+05 45.00000 0.3 8.0
10 20 1 1 10 1 .01
1 1 1
2 30.

```

3				60.	
4	1	1			40.
5				30.	40.
6				60.	40.
1	4	5	0	1	.25
2	4	2	0	2	.20
3	1	5	0	2	.20
4	1	2	0	1	.25
5	2	5	0	1	.20
6	5	6	0	1	.25
7	5	3	0	2	.20
8	2	6	0	2	.20
9	2	3	0	1	.25
10	3	6	0	1	.20
3			-10.		
MIT	SHEAR	LAG	PROBLEM	CONFIGURATION	1
49	78	2	1	22	30
0.10200000E+08	52000.			0.3	5.
10	20	1	1	1	0.01
1	1	1	0.0		
2		1	0.5		
3		1	1.0		
4		1	1.5		
5		1	2.0		
6	1		0.0	0.5	
7			0.5	0.5	
8			1.0	0.5	
9			1.5	0.5	
10			2.0	0.5	
11		1	3.0		
12		1	4.0		
13		1	5.0		
14	1		0.0	1.0	
15			0.5	1.0	
16			1.0	1.0	
17			1.5	1.0	
18			2.0	1.0	
19			3.0	1.0	
20			4.0	1.0	
21			5.0	1.0	
22	1		0.0	2.0	
23			0.5	2.0	
24			1.0	2.0	
25			2.0	2.0	
26			3.0	2.0	
27			4.0	2.0	
28			5.0	2.0	
29	1		0.0	3.0	
30			0.5	3.0	
31			1.0	3.0	
32			2.0	3.0	
33			3.0	3.0	
34			4.0	3.0	
35			5.0	3.0	
36	1		0.0	4.0	
37			0.5	4.0	
38			1.0	4.0	
39			2.0	4.0	
40			3.0	4.0	
41			4.0	4.0	
42			5.0	4.0	
43	1		0.0	5.0	

44			0.5	5.0
45			1.0	5.0
46			2.0	5.0
47			3.0	5.0
48			4.0	5.0
49			5.0	5.0
1	1	2	6	1 .08
2	2	6	7	1 .08
3	2	3	7	1 .08
4	3	7	8	1 .08
5	3	4	8	1 .08
6	4	8	9	1 .08
7	4	5	9	1 .08
8	5	9	10	1 .08
9	5	10	11	1 .08
10	10	11	19	1 .08
11	11	12	19	1 .08
12	12	19	20	1 .08
13	12	13	20	1 .08
14	13	20	21	1 .08
15	6	7	14	1 .08
16	7	14	15	1 .08
17	7	8	15	1 .08
18	8	15	16	1 .08
19	8	9	16	1 .08
20	9	16	17	1 .08
21	9	10	17	1 .08
22	10	17	18	1 .08
23	10	18	19	1 .08
24	14	15	22	1 .08
25	15	22	23	1 .08
26	15	16	23	1 .08
27	16	23	24	1 .08
28	16	17	24	1 .08
29	17	24	25	1 .08
30	17	18	25	1 .08
31	18	19	25	1 .08
32	19	25	26	1 .08
33	19	20	26	1 .08
34	20	26	27	1 .08
35	20	21	27	1 .08
36	21	27	28	1 .08
37	22	23	29	1 .08
38	23	29	30	1 .08
39	23	24	30	1 .08
40	24	30	31	1 .08
41	24	25	31	1 .08
42	25	31	32	1 .08
43	25	26	32	1 .08
44	26	32	33	1 .08
45	26	27	33	1 .08
46	27	33	34	1 .08
47	27	28	34	1 .08
48	28	34	35	1 .08
49	29	30	36	1 .08
50	30	36	37	1 .08
51	30	31	37	1 .08
52	31	37	38	1 .08
53	31	32	38	1 .08
54	32	38	39	1 .08
55	33	39	32	1 .08
56	33	39	40	1 .08

57	33	34	40	1 .08
58	34	40	41	1 .08
59	34	35	41	1 .08
60	35	41	42	1 .08
61	36	37	43	1 .08
62	37	43	44	1 .08
63	37	38	44	1 .08
64	38	44	45	1 .08
65	38	39	45	1 .08
66	39	45	46	1 .08
67	39	40	46	1 .08
68	40	46	47	1 .08
69	40	41	47	1 .08
70	41	47	48	1 .08
71	41	42	48	1 .08
72	42	48	49	1 .08
73	1	6		10.0452
74	6	14		10.0754
75	14	22		10.15
76	22	29		10.25
77	29	36		10.35
78	36	43		10.45
43			10000.	

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<p>A computer program is presented for the small strain analysis of plane structures in the strain hardening elastic-plastic range. The finite element displacement method is used to perform the linear analyses in the iterative scheme. Bar and constant strain isotropic plane stress triangles are available for use in constructing structural idealizations. The use of ten different sets of material properties, three different material laws, and incremental proportional loading are available as options. Good correlation is shown with available test data and theoretical solutions.</p> <p>The distribution of this abstract is unlimited.</p>			

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